ELECTRICAL

AND

MAGNETIC CALCULATIONS

FOR THE USE OF

Electrical Engineers and Artisans, Teachers, Students, and all others interested in the Theory and Application of Electricity and Magnetism

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BY

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PREFACE.

The following pages, both in plan and material, are the outgrowth of several years of experience in teaching young men the rudiments of electricity. A large part of the matter, in fact, was prepared expressly as an introduction to a course in electrical engineering; since there was nothing published covering the topics found desirable, and making use of the method herein employed.

A multiplicity of wordy rules and unexplained constants arbitrarily set down burden the memory unnecessarily, are often unintelligible to the reader, and are at best very clumsy tools with which to work. In the present volume, on the other hand, several processes are brought together wherever possible under a single broad principle, which is then expressed by means of a formula. The treatment in this respect aims to be educational. Through a step by step process principles and formulæ are evolved from facts and principles already understood. After the law has been clearly developed, and has been given the most concise, easily remembered, and convenient working form, the method of induction gives way to that of deduction. A series of examples are then worked out illustrating the application of the principle, and giving familiarity with its processes. At the end of the chapters are also lists of original problems for drill in the mastery of the principles and their application.

It is hoped that the great body of artisans in all the departments of electrical engineering practice will find in these pages an invaluable aid in their efforts to acquire a better working knowledge of the principles underlying their profession. In a first reading, perhaps the original problems may be passed over; also the chapter on the relation of heat and chemical energy, and possibly a portion of alternating currents, especially that of long-distance transmission circuits. Each individual's taste and particular line of work will largely guide him in his selection of subjects for special investigation. A careful study of this little volume, even omitting certain portions as suggested, will be an excellent schooling for those busily engaged in the various electrical processes.

It will also be a helpful companion to electrical engineers, superintendents, and all those in the more responsible positions in engineering work. They will find it valuable in its development of the rules and formulæ employed in their profession and as a handy reference for the methods of application of these rules and formulæ to practical engineering problems.

Teachers in schools and colleges which devote some time to the subject of physics may find here a vast number of examples for class use in teaching electricity, the most important branch of physics. It will be particularly useful to teachers in colleges and technical schools which make a special feature of electricity, either as a reference book of formulæ and examples, or as a text-book for class drill in those topics treated. Selections may be made of the topics best suited to particular needs, if not all are available.

In the preparation of this book frequent use has been made of competent authorities. Most of these have been mentioned in the body of the work, or in the marginal references.

Mistakes in a book of this nature are inevitable, however carefully the proof has been read, and the author will appreciate any corrections sent to him.

This opportunity is taken of thanking our friend and professor, W. M. Stine, Professor of Engineering in Swarthmore College, for examination of the manuscript and valuable suggestions in connection with its publication. Acknowledgments are also due to Professor W. B. Bentley, Department of Chemistry, Ohio University, for assistance in reading the proof, and to Messrs. F. H. Super and N. R. Cunius, Assistants in the Department of Physics and Electricity, Ohio University, for the preparation of the diagrams.

OHIO UNIVERSITY,
Jan. 1, 1902.

PREFACE TO THE SECOND EDITION.

THE author has been gratified by the many appreciative reviews and notices published in reference to the first edition of this volume. It is not a treatise on Electrical Engineering, but, as its title indicates, attempts to set forth the methods of solution of those problems in Electricity and Magnetism which are of most importance to the electrical engineer and the teacher. As such it has been most kindly received.

Coming so soon after the publication of the first edition perhaps not all the errors have been discovered and corrected, and but few changes are considered necessary, otherwise.

The chapter on Alternating Currents has been moved to the end of the book, and Chapter XIII., Alternating Current Distribution, formed from the portion of Chapter VI., relating to Alternating Currents, added. These modifications will no doubt improve the usefulness of the book.

OHIO UNIVERSITY,
Oct. 1, 1902.

PREFACE TO THE FOURTH EDITION

The present edition of this text has been improved in the following particulars: A considerable number of errors in the former edition have been noted and corrected in this; some terms made obsolete by common usage of a substitute, such as "duty" and "weber," have been changed to the modern terms; in Chapter V constants are suggested by whose use the rules for wiring, applicable to carbon lamps, may also be adapted to tungsten lamps now in common use; also it is suggested on pages 143, 147 and 199 that the original problems in Chapters VIII, X and XI, in which magnetomotive force or ampere-turns are required, be worked out by the use of the curves on page 229, or by the use of the tables given on page 298. From the curves the value of H corresponding to any value of B is obtained for each portion of the magnetic circuit; each H (gilberts per centimeter) is then multiplied by the corresponding length in centimeters, giving the magnetomotive force, whence the sum of all the partial magnetomotive forces multiplied by eight-tenths will give the total ampere-turns required; that is, *ampere-turns* = 0.8 M. From the tables on page 298 the "ampere-turns per centimeter" are obtained direct — $\frac{nI}{l}$ — whence the ampere-turns for each portion and the total for the magnetic circuit may readily be computed. These

methods will give results only approximately the same as those obtained by the more cumbersome reluctance method, for reasons which will at once be apparent. Their convenience and practicability recommend their general use for all such computations.

It is to be hoped that those accustomed to the use of this book, as well as others who may use it, will find it more satisfactory because of the changes and suggestions made in this edition.

OHIO UNIVERSITY, ATHENS, OHIO,
Jan. 2, 1913.

ELECTRICAL AND MAGNETIC CALCULATIONS.

I.

EXPLANATION OF UNITS.

1. **Units.** — In every form of measurement certain *units* are necessary as standards of comparison. A quantity of any kind is measured when the number of the units it contains is determined. Thus a bin of wheat is measured when the number of bushels it contains is found, the bushel being the *unit.* The quantity of electrical flow is measured by expressing the number of *coulombs* in it. The expression of a quantity, therefore, includes the name of the unit employed, preceded by the number of units; as 40 volts, or 5 amperes.

2. **Fundamental and Derived Units.** — The *fundamental* units of any system are those which are basal, not derived from any other units. There are considered generally three fundamental units, — the unit of *time*, the unit of *mass*, and the unit of *length.* In one system these are respectively, — the second, the gram, and the centimeter. These are the units used for all scientific work. Other units built up by combining these units are called *derived* units. Such are units of area involving the sec-

ond power of a length, l^2; also units of volume being proportional to l^3. Likewise the *watt* is a derived unit, being a product; its factors are an electromotive force and a current, or $E \times I$, each of which in turn is derived.

3. **Absolute Units.** — If the fundamental units mentioned above be used, the system of *absolute units* based upon them is called the C.G.S. system, or the centimeter-gram-second system, from the names of the fundamental units of the system. Thus the *dyne* is the absolute unit of force, being that force which, acting upon one gram for one second, will give it a velocity of one centimeter per second. The *erg* is the absolute unit of work. It is the work done when one dyne acts through a distance of one centimeter.

It is to be observed that a unit is defined by unit conditions; that is, by the units of the elements upon which the unit under definition is based. To illustrate, work involves two, and only two elements; namely, force and distance. Hence, unit work will be defined by unity in these elements. The *erg* is the work of *one dyne* through *one centimeter.*

4. **The Basis of the Fundamental Units.** — (*a*) Originally the *meter* was intended to be the ten-millionth part of the earth-quadrant through the meridian of Paris, measured from the equator to the north pole. For practical purposes, the French government adopted a platinum standard between whose ends at 0° C. the distance should be one meter. This is known as the "Metre des Archives." It was made by Borda in accordance with a government decree passed in 1795. In 1866, the United

States government, by Act of Congress, defined the meter to be 39.37 inches. The *centimeter* is the one-hundredth part of the meter; the *millimeter* is the tenth part of the centimeter, or the one-thousandth part of the meter. For larger measures the kilometer is employed, and is equal to 1000 meters.

(*b*) Mass is the quantity of matter in a body, while its weight is the amount of the earth's force of gravity upon the mass. The mass does not vary, but weight varies from place to place due to changes in the force of gravity. Calling the force of gravity g, representing mass by m, and weight by w, this relation is expressed by

$$w = mg. \qquad (1)$$

Weight is therefore a function both of mass and the force of gravity. In the C.G.S. system the unit of mass is theoretically that of a cubic centimeter of distilled water at the temperature of its greatest density, 4° C., and is called the *gram*. As a practical standard, however, it is the one-thousandth part of a mass of platinum preserved in the archives of Paris, called the "Kilogram des Archives." The reason for using the larger material standards for length and mass is obvious. The *kilogram* has the value in English measure of 2.20462 pounds, or approximately $2\frac{1}{5}$ lbs.

(*c*) The *second* is the $\frac{1}{86400}$ part of the average length of all the solar days taken throughout the year. A solar day is the interval between two successive transits of the sun's center across the meridian of a place. These days vary in length on account of the unequal velocities of the

earth in its orbit from day to day. Hence the average of all is taken throughout the year.

5. **Magnetic Units.** — (**a**) The **Strength** of **Pole** of a magnet is determined by the force it is capable of exerting on another pole at a given distance. If m be the strength of one pole and m' that of another at a distance of d, then the force of attraction or repulsion, as the case may be, is expressed by the formula,

$$F = q\frac{mm'}{d^2}. \tag{2}$$

in which q is a constant. The force, therefore, varies directly as the strength of the poles and inversely as the square of their distance apart. A *unit pole* is one such as to repel a similar one at a distance of one centimeter with a force of one dyne.

(**b**) **Magnetic Field.** Any space traversed by magnetic forces is called a *magnetic field.* The space between the poles of a horse-shoe or other magnet is conceived to be traversed by magnetic *lines of force;* that is, imaginary lines along which magnetic attraction or repulsion takes place. If the force is one dyne per square centimeter of the surface normal to the direction of the lines of force, we say the field has an intensity of one; if the force be 10 dynes per square centimeter, the intensity is 10, or 10 lines of force per square centimeter, etc. Intensity of field is represented by H. If a pole of strength m be put into a field whose intensity is H, the force tending to push it along the lines is

$$F = Hm. \tag{3}$$

(c) **Magnetic Moment.** The *magnetic moment* of a magnet is the product of its strength of pole by its length, or

$$M = ml. \tag{4}$$

Suppose a slim magnet of length l be placed at right angles to the lines of force of a field of intensity H, then the moment of the couple tending to swing the magnet around into parallelism with the lines of force will be

$$F = Hml = HM. \tag{5}$$

(d) **Intensity of Magnetization.** The quotient obtained by dividing the magnetic moment by the volume of the magnet is called the *intensity of magnetization.*

$$I = \frac{M}{V} = \frac{ml}{sl} = \frac{m}{s}. \tag{6}$$

In other words, it may be defined as the strength of pole per unit surface of cross section.

(e) **Magnetic Induction.** When a piece of soft iron is placed parallel with the lines of force of a magnetic field, the lines concentrate within the bar, which becomes a magnet by *induction.* The end at which the lines enter becomes a south pole, the end from which they leave a north pole. The iron offers a path of less resistance to the magnetic lines than air does. This is expressed by saying that iron has a smaller reluctivity, or a greater *permeability*, than air. When speaking of complete magnetic circuits the terms *reluctance* and *permeance* are used instead of the above, which apply to specific substances. Air is taken as the standard of permeabilities, its own being therefore

unity. The number of lines of force passing through a given length of different substances is proportional to their respective permeabilities, the areas of cross section being 1 square centimeter. If H represents the intensity of the lines in air, and the intensity of induction in a piece of iron put into the field H be expressed by B, then under the conditions the permeability of the iron will be

$$\mu = \frac{B}{H}.$$

from which

$$B = \mu H. \tag{7}$$

If the bar, say, steel, be already magnetized to intensity I, when placed in the field H, the induction will become

$$B = H + 4\pi I. \tag{8}$$

6. **Electrical Units.** — There are two systems of electrical units: (1) units founded on the force of attraction of unlike or repulsion of like charges of electricity, constituting the *electrostatic* system; (2) units based on considerations of the magnetic field produced by a current of electricity flowing in a wire, constituting the *electromagnetic* system. All our practical electrical units are derived from the latter system, which is, therefore, the only one that will be considered.

(a) **The Unit of Current.** The absolute unit of current is defined as that *rate of flow in a conductor bent into the form of a ring whose radius is one centimeter, which will exert a force of one dyne at the center for each centimeter of the arc.* Hence, the radius being unity, the whole circumference, $2\pi r$, will be 6.28 cm., and the total force

exerted at the center 6.28 dynes. This unit is the absolute *ampere.* Suppose, now, a loop of any radius r be made to carry any number of absolute amperes I; the force at the center of the loop in dynes will be

$$F = \frac{2\pi r I}{r^2} = \frac{2\pi I}{r} = 6.28 \frac{I}{r}. \tag{9}$$

If the circular coil have n turns instead of one, the force becomes

$$F = \frac{2\pi n I}{r}. \tag{10}$$

In case the wire is wound in a long solenoid whose length l is great compared with its radius, the force inside it will be

$$F = \frac{4\pi n I}{l}. \tag{11}$$

This unit is large, and for practical purposes the ampere is chosen equal to $\frac{1}{10}$ of the above unit, so that if one *practical ampere* flow in the loop as described the force at the center due to the whole circumference will be $6.28 \times 10^{-1} = 0.628$ dyne.

The Chamber of Delegates of the International Congress of Electricians which convened in Chicago, Aug. 21, 1893 (World's Fair), defined the unit of current as follows:

As the unit of current, the *international ampere* is recommended to be adopted which is one-tenth of the unit of current of the C.G.S. system of electromagnetic units, and which is represented sufficiently

well for practical use by the unvarying current which, when passed through a solution of nitrate of silver in water, in accordance with accompanying specifications, deposits silver at the rate of 0.001118 gram per second.

If the solution be copper sulphate the international ampere will deposit copper at the rate of 0.0003284 gram per second under specific conditions. These constants, 0.001118 and 0.0003284, are called the *electrochemical equivalents* of silver and copper, respectively. Sub-divisions of the ampere are often used for the measurement of very small currents; as the *milliampere*, or the $\frac{1}{1000}$ of the ampere, and the *microampere*, or the $\frac{1}{1000000}$ of the ampere.

(b) **The Unit of Quantity.** The quantity of electricity conveyed is expressed by the product of the current and the time flowing in seconds. *Unit quantity will be transferred when one ampere flows for one second.* This is called the *coulomb* and is equal to $10^{-1} \times 1$, or 10^{-1} C.G.S. units. Suppose 5 amperes flow for 4 seconds; $Q = 5 \times 4 = 20$ coulombs, or $20 \times 10^{-1} = 2$ C.G.S. units of quantity. *Microcoulombs* are used to express small quantities.

(c) **The Unit of Electromotive Force.** The general definition of force is that which tends to produce or modify motion. By analogy, one would say that electromotive force is that which tends to move or transfer a quantity of electricity. It is analogous to pressure produced by a head of water, as in a standpipe. If a tap be opened somewhere in the street water-main there will be a flow through the pipe because the pressure at the tap is less than that at the standpipe. Let E_1 be the pressure at the

latter, and E_2 the pressure at the tap. The quantity flowing out is proportional to $E_1 - E_2 = E$. This difference of pressure, which may be represented simply by E, may be called the *watermotive* force. Similar conditions exist in the phenomenon of electromotive force, or electric pressure. All that the battery or dynamo does is to set up a difference of potential, or difference of electric pressure between two points, called electromotive force E. The phenomenon of the equalization of this difference in a circuit is what we mean by current flow. Now, if the opposing forces, or resistance, be unity, the unit of E.M.F. is that which will produce unit flow of current through the circuit.

The absolute unit of force being very small, the *practical* unit of E.M.F. is chosen equal to 100,000,000 or 10^8 C.G.S. units, and is called the *volt*. The International Congress, mentioned before, recommended for adoption,

As the unit of electromotive force, the *international volt*, which is the E.M.F. that steadily applied to a conductor whose resistance is one *international ohm*, will produce a current of one *international ampere*, and which is represented sufficiently well for practical use by $\frac{1000}{1434}$ of the E.M.F. between the poles of the voltaic cell, known as Clark's Cell, at a temperature of 15° C., and prepared according to specification.

The Clark cell has therefore 1.434 international volts of E.M.F. The Carhart cell, which is much superior to the old Clark, is now much used in this country as a standard. It has an E.M.F. of 1.44 volts at 15° C.

One *volt* will be generated in a conductor which is moved across a magnetic field so as to cut the lines of force at the rate of 10^8 per second.

(d) **The Unit of Resistance.** *Resistance* to the flow of a current is of the nature of an opposing force. The *unit* of *resistance* is such that one volt of E.M.F. will cause current to flow through it at the rate of one ampere per second. The *practical* unit is chosen equal to 10^9 C.G.S. units of force and is called the *ohm*. The adopted *international ohm*

Is based upon the ohm equal to 10^9 units of resistance of the C.G.S. system of electromagnetic units, and is represented sufficiently well for practical use by the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross sectional area, and of the length 106.3 centimeters.

For practical standards, coils of constantan or manganin wire are wound on spools, standardized and placed in a suitable box, the coils being connected to brass bars on the top, so that by using plugs any resistance within the capacity of the box can be obtained. The *megohm*, or 1,000,000 ohms, is used in the measurement of high resistances, such as the resistance of insulators. For very small resistances the *microhm*, or $\frac{1}{1000000}$ ohm is used.

(e) **The Unit of Capacity.** If one coulomb of electricity be stored in a recipient, for instance, in a coil of insulated wire, or in a system of flat parallel conductors, adjacent ones insulated from each other, called a condenser, and if this quantity tends to escape with an E.M.F. of one volt, the *capacity* of the recipient or condenser is unity. This unit is the *farad*. Expressed in another way, it is the capacity such that one volt will store in it one coulomb of electricity. For all ordinary capacities the *microfarad*, or

$\frac{1}{1000000}$ of the farad is used, the farad being very large, 10^{-9} C.G.S. units.

The capacity of the earth is $\frac{636}{1000000}$ farad, or 636 microfarads. A Leyden jar with a total tinfoil surface of 1 square meter, and glass 1 millimeter in thickness, has a capacity of $\frac{1}{55}$ microfarad.

(f) **The Unit of Power.** The unit of electrical power is called the *watt*, and is the energy required to move one ampere per second through one ohm resistance. In other words, one volt of E.M.F. delivering one ampere of current per second represents a power of one *watt*. The watt is equal to 10^7 ergs per second of C.G.S. units. The *horse-power* is 33,000 foot-pounds of mechanical work per minute. The *watt* is $\frac{1}{746}$ of the horse-power, and 1 H.P. = 746 watts.

(g) **The Unit of Work.** The unit of work is done when one watt of energy is expended per second, and is called the *joule*, which is also equal to 10^7 ergs. Joules of work are obtained by multiplying watts of power, or energy, by seconds of time.

(h) **The Unit of Induction.** The *induction* is unity when the E.M.F. induced is one international volt, while the inducing current varies at the rate of one international ampere per second. This unit is called the *henry*, and is 10^9 C.G.S units.

These international electrical units were legalized by Act of Congress, approved by the President, July 12, 1894, so that they take their place with other standards of weights and measures.

II.

RELATION OF QUANTITIES.

7. **Ohm's Law.** — The simplest perhaps, and yet the most important relation of electrical or magnetic quantities to each other is that expressed in Ohm's law. This law expresses the relation of E.M.F., current, and resistance. Putting I for current, E for E.M.F., and R for resistance, the law is expressed by the formula,

$$I = \frac{E}{R}. \qquad (12)$$

Hence, current is obtained by dividing electromotive force by resistance. From the above formula, by simple transposition, or by the rules of simple division, we obtain $E = IR$, and $R = \frac{E}{I}$. In words, these mean that *the E.M.F. is equal to the product of current and resistance*, and that *resistance is equal to the quotient of E.M.F. by current.*

Example. — How much current in amperes will flow through an incandescent lamp whose resistance is 200 ohms, when an E.M.F. of 110 volts is applied to it?

Solution. — $I = \frac{E}{R} = \frac{110}{200} = 0.55$ ampere.

Example. — A battery has a resistance r of 3 ohms; what is its E.M.F. if it causes a current of 0.05 amperes to flow through an external resistance equal to 60 ohms?

SOLUTION. — $E = I(R + r) = 0.05\,(60 + 3) = 3.15$ volts.

EXAMPLE. — Find the resistance of an arc street lamp when a voltage of 50 is required to send 7 amperes through it.

SOLUTION. — $$R = \frac{E}{I} = \frac{50}{7} = 7\frac{1}{7} \text{ ohms.}$$

Expressed in C.G.S. units, $E = 10^8$, $R = 10^9$, and $I = \frac{E}{R} = \frac{10^8}{10^9} = 10^{-1}$ or $\frac{1}{10}$. That is, the ampere is the $\frac{1}{10}$ of the C.G.S. electromagnetic unit of current.

8. Quantity, Electromotive Force and Capacity. — EXAMPLE. — How many coulombs of electricity will flow through an incandescent lamp whose resistance is 200 ohms, when an E.M.F. of 110 volts is applied to it for 2 seconds?

SOLUTION. — $Q = I \times t = \frac{E}{R} \times t = \frac{110}{200} \times 2 = 1.1$ coulombs.

EXAMPLE. — A glass plate has a square of tinfoil pasted on each side, thus forming a storehouse or condenser. A battery whose E.M.F. is 10 volts is connected to both sides by wires, and thus gives it a charge of 0.000000001 coulomb = 0.001 microcoulomb: what is the capacity of the receptacle?

SOLUTION. — From obvious considerations the quantity stored will depend on the pressure applied and the capacity of the storehouse: that is

$$Q = EC. \tag{13}$$

From this

$$C = \frac{Q}{E} = \frac{0.000000001}{10} = 0.0000000001 \text{ farad}$$
$$= 0.0001 \text{ microfarad.}$$

Example. — How many volts will be required to charge a condenser whose capacity is 0.5 microfarad with 2.5 microcoulombs of electricity?

Solution. — From equation (13) $E = \frac{Q}{C}$. Hence, in this case, $E = \frac{2.5}{0.5} = 5$ volts, the required E.M.F. Or, since 2.5 microcoulombs = 0.0000025 coulomb, and 0.5 microfarad = 0.0000005 farad, $E = \frac{0.0000025}{0.0000005} = 5$ volts, as before.

Example. — Find the quantity in microcoulombs which a cell of 1.8 volts E.M.F. will store in a condenser whose capacity is 0.1 microfarad.

Solution. — $Q = EC = 1.8 \times 0.1 = 0.18$ microcoulomb.

Example. — Find the value of the last answer in absolute C.G.S. units.

Solution. — The coulomb $= 10^{-1}$ C.G.S. units of quantity. 0.18 microcoulomb = 0.00000018 coulombs = $0.00000018 \times 10^{-1}$ C.G.S. units.

9. **Power and Work.** — Example. — A storage battery has an E.M.F. of 26 volts; assuming the internal resistance r to be 2 ohms, and the external resistance R to be 50 ohms, how many joules of work will it do in 30 minutes?

Solution. — 30 min. = 1800 sec. $I = \frac{E}{R} = \frac{26}{2 + 50} = \frac{1}{2}$ ampere. Watts $= E \times I = 26 \times \frac{1}{2} = 13$. Joules = watts × seconds = 13 × 1800 = 23,400 joules.

Or, $$\text{Watts} = \frac{E^2}{R+r} = \frac{\overline{26}^2}{52} = 13\text{; and}$$

$$1800 \times 13 = 23400 \text{ joules.}$$

Example. — If the indicator shows that an engine is developing 30 H.P., what is the output of the dynamo to which it is attached, neglecting all losses in both engine and dynamo?

Solution. — 30 H.P. = 30 × 746 = 22,380 watts = 22.38 kilowatts, or K.W.

Example. — Suppose the above dynamo supplies 400 lamps at 110 volts: how much current must each lamp require, and how much work is done in each per night of 10 hours?

Solution. — 22,380 watts ÷ 110 volts = 203.45 amperes. This amount is for 400 lamps. Hence each will receive 203.45 ÷ 400 = 0.508 ampere. 110 volts × 0.508 = 55.88 watts per lamp. 10 hours = 10 × 60 × 60 = 36,000 seconds. Hence the total work done = 55.88 × 36,000 = 2,011,320 joules per lamp.

10. The Magnetic Relations of Current. — Example. — Let a circular loop of wire having a radius r of 5 cm. carry 10 C.G.S. amperes of current; how much force in dynes will be exerted at its center?

Solution. — $$F = \frac{2\pi I}{r} = 6.28\frac{I}{r} = 6.28 \times \frac{10}{5}$$
$$= 12.56 \text{ dynes.}$$ See equation (9).

Example. — How many international amperes will be

required in a circular coil of 20 turns, radius of coil 10 cm., to produce a force of 125.6 dynes at its center?

SOLUTION.— From (10), $F = 6.28 \frac{nI}{r}$ and $I = \frac{rF}{6.28\,n}$.

Therefore $I = \frac{10 \times 125.6}{6.28 \times 20} = 10$ C.G.S. units. But the international ampere is 10^{-1} C.G.S. units. Therefore 10 C.G.S. amperes $= 10 \div 10^{-1} = 100$ amperes.

EXAMPLE. — How many turns of wire will be necessary, and on what radius wound, to produce a force of 125.6 dynes at the center when carrying 1 ampere?

SOLUTION.— $$F = \frac{2\,\pi n I}{10\,r} = 125.6.$$

Hence $$\frac{n}{r} = \frac{125.6 \times 10}{2\,\pi I} = \frac{1256}{6.28 \times 1} = 200.$$

An indefinite number of answers is obviously possible. Within reason, however, a comparatively small number of solutions is permissible. For instance, make $n = 400$; then $r = 2$ cm. Or say $n = 800$, whence $r = 4$ cm. Again, n may be 1000 and $r = 5.0$ cm.

EXAMPLE. — How many dynes of force will be exerted inside a solenoid, or long coil, whose length is 20 cm., and the number of whose turns is 100, when it carries 10 amperes of current?

SOLUTION.— From (11),

$$F = \frac{4\,\pi n I}{l} = \left(12.56 \times 100 \times \frac{10}{10}\right) \div 20 = 62.8 \text{ dynes.}$$

11. **Electrolytic Effects.** — EXAMPLE. — How many amperes of current have been flowing for 30 minutes when

the amount of silver deposited by it upon the negative plate, or cathode, is found to be 0.0040248 gram?

Solution. — The current multiplied by the amount that 1 ampere in 1 second deposits, and this product by the time in seconds, gives the total weight deposited. Briefly,

$$W = Izt, \tag{14}$$

in which W is the weight in grams deposited on the cathode; z is the weight deposited per second by 1 ampere; and t is the time in seconds that the current is passing through the solution of silver nitrate, $AgNO_3$. In this problem, $t = 30 \times 60 = 1800$ seconds; $z = 0.001118$ gram per ampere per second, or the *electrochemical equivalent* of silver; $w = 0.0040248$ gram. Hence

$$I = \frac{W}{zt} = \frac{0.0040248}{0.001118 \times 1800} = 0.002 \text{ amp.}$$

Example. — How long must 1 ampere flow through a solution of copper sulphate, $CuSO_4$, with copper electrodes, cathode, and anode, to deposit 0.3284 gram of copper?

Solution. — From (14),

$$t = \frac{W}{Iz} = \frac{0.3284}{1 \times 0.0003284} = 1000 \text{ sec.} = 16 \text{ min. } 40 \text{ sec.}$$

The electrochemical equivalent of copper is 0.0003284 gram per ampere per second.

Example. — How much water, H_2O, will be decomposed, that is, separated into its elements of O and H, by 5 amperes flowing between platinum electrodes for 60 minutes?

Solution. — The electrochemical equivalent of H = 0.00001038 gram; of O = 0.00008283 gram. Hence 1 ampere will decompose in 1 second, 0.00001038 + 0.00008283 = 0.00009321 gram of water. Therefore, (14), $W = Izt = 5 \times 0.00009321 \times 3600 = 1.67778$ grams = 1.67 cubic centimeters at 4° C.

12. **Original Problems.** — 1. What must be the external resistance of a circuit so that a battery of 1.8 volts E.M.F. and $\frac{1}{5}$ ohm internal resistance will send 1.5 amperes through it?

$R = 1$ ohm.

2. An incandescent lamp has a resistance, when hot, of about 220 ohms, and requires $\frac{1}{2}$ ampere of current. If the dynamo has a resistance of $\frac{1}{10}$ ohm, how many ohms may the lead wires have if the voltage of the dynamo is 111?

$R = 1.9$ ohms.

3. What must be the E.M.F. of a generator to supply 40 lamps each requiring $\frac{1}{2}$ ampere, the resistance of the generator being 0.1 ohm, so that each lamp, resistance 220 ohms, shall get its full current, assuming the wires to have a resistance of 0.5 ohm?

$E = 122$ volts.

4. A storage battery consists of 10 cells in series, each giving 2 volts E.M.F. and having $\frac{1}{4}$ ohm internal resistance; the external circuit consists of two resistance coils in series, one of $5\frac{1}{2}$ ohms, the other 8 ohms; how many amperes of current will flow in the circuit?

$I = 1.25$ amperes.

5. When an E.M.F. of 110 volts is applied to a bank of 55 incandescent lamps in parallel, 27.5 amperes flow in the circuit; how many ohms resistance in the bank? How does this compare with the resistance of a single lamp? What does this last illustrate?

$R = 4$ ohms.
One lamp has $\frac{220}{4} = 55$ times as much as 55 lamps.

6. How many coulombs would be used in the bank, in a 6 hours' run, and how much must be charged per ampere-hour in order to make the income equivalent to 40¢ per lamp per month, the month consisting of 30 days, and 6 hours' run each day?

$Q = 594{,}000$ coulombs.
Price $= 0.44$¢ per ampere-hour.

7. A cell whose E.M.F. is 1 volt is used to charge a condenser which is afterwards discharged through a circuit in which is a milliamperemeter. The discharge required 1 second and the milliammeter read $\frac{1}{10}$ milliampere. Find the capacity of the condenser in microfarads.

$C = 100$ mf.

8. What must be the E.M.F. of the charging battery when it is required that a condenser whose capacity is $\frac{1}{3}$ microfarad, on discharge gives 0.000002 coulomb through the ballistic galvanometer?

$E = 6$ volts.

9. There are 500 lamps, 110 volts, 220 ohms each in a certain building, and they burn 5 hours out of each 24. How much must the company furnishing the power

charge per watt-hour, meter rates, in order that it may realize the equivalent of 50¢ per lamp per month, flat rate, assuming 30 days per month?

Price 0.0061¢ per watt-hour.

10. A dynamo generates 116 volts, of which 110 volts are required in the lamps, the other 6 being necessary to pass the current through the machine and line. The lamps have a resistance each of 220 ohms, and there are 40 of them in parallel. What per cent of the watts is spent in the lamps, and what per cent as waste in the machine and line?

Per cent in lamps = 94.8.
Per cent in waste = 5.2.

11. Find the H.P. of the machine in problem 10, and the H.P. used in the lamps.

H.P. of machine = 3.1.
H.P. of lamps = 3 –.

12. Reduce the machine energy in 10 to C.G.S. values and to joules of electrical work.

Joules per second = 2320.
Ergs per second = 2320×10^7.

13. How many turns of wire will be necessary to make a solenoid 2 cm. diameter and 50 cm. long so that 5 amperes will produce a field within which the force shall be 62.8 dynes?

$n = 500$ turns.

14. Determine the amount of current that must flow in a circuit in which is a silver voltameter so that the

cathode which weighs before passing the current 26.7571 grams shall weigh, after passing the current 1 hour, 26.9128.

$I = 0.03868$ ampere.

15. The above current passed through a tangent galvanometer whose deflection was $53.5^\circ = \theta$. The law of the tangent galvanometer is $I = K \tan \theta$, where I is the current, K is the galvanometer constant, or the current necessary to cause a deflection whose tangent is 1. Find K.

$K = 0.0286$ ampere.

16. Find the constant K of a small portable galvanometer in series with a copper voltameter, when the following observations were taken: weight of cathode before test, 4.82465 grams; after passing current 30 minutes, weight was 4.831 grams; deflection, $\theta = 53^\circ$.

$K = 0.00811$.

III.

GENERAL LAWS OF RESISTANCE.

13. **The Relation of Resistance to Length and Area.** — Experiment shows that the electrical resistance of a conductor varies directly with its length, the kind of material, and inversely with its cross sectional area. Analogy to the flow of water in iron pipes teaches us the same relation. The longer the pipe, the greater the resistance to flow; the rougher or the more crooked the pipe, the greater the resistance; the larger the pipe, also, the less the resistance, and the greater the flow. Stating these relations by means of a formula, we have

$$R = K\frac{l}{a}. \tag{15}$$

in which R is the resistance, l the length, a the area of cross section, and K is a constant depending on the material of the wire, and means the resistance in ohms of unit dimensions of the material. If the length is expressed in feet and the area in circular mils, as it is usually in estimating the resistance of wires, then K is the resistance of a *mil-foot;* that is, the resistance of a piece 1 foot long and 1 mil in diameter, or 1 circular mil in cross sectional area. A *mil* is the $\frac{1}{1000}$ of an inch, and *circular mils* are obtained by simply squaring the diameter in mils. *Square mils* are then obtained, if necessary, by multiplying circular mils by 0.7854.

In comparing two wires in order to obtain any one of the four quantities in (15), it will be more convenient to write down the formula for each wire, using subscripts to denote wire No. 1 and wire No. 2, then after substituting the given terms in each case, take the ratio of the two equations. For example, for wire No. 1:

$R_1 = K_1 \frac{l_1}{a_1}$ (1), and for wire No. 2 likewise,

$R_2 = K_2 \frac{l_2}{a_2}$ (2). Or if areas are to be expressed in circular mils as is usually better,

$R_1 = K_1 \frac{l_1}{d_1^2}$ (1) for wire No. 1, and

$R_2 = K_2 \frac{l_2}{d_2^2}$ (2) for wire No. 2. Now taking the ratio of (1) and (2), we have

$$\frac{R_1}{R_2} = \frac{K_1}{K_2} \times \frac{l_1}{l_2} \times \frac{d_2^2}{d_1^2} \quad (3).$$

EXAMPLE. — A length of 1000 feet of wire 95 mils in diameter has a resistance of 1.15 ohms; what is the diameter of a wire of the same material whose resistance is 5.045 ohms for 500 feet?

SOLUTION. — $R_1 = 1.15$ ohms; $R_2 = 5.045$ ohms.
$l_1 = 1000$ ft.; $l_2 = 500$ ft.
$K_1 = K_2$; $d_1 = 95$ mils; $d_2 = ?$

Using (3) above, and making the proper substitutions,

$\frac{1.15}{5.045} = \frac{1000}{500} \times \frac{K_2}{K_2} \times \frac{d_2^2}{\overline{95}^2}$. Whence, transposing,

$$d_2^2 = \frac{\overline{95}^2 \times 1.15}{2 \times 5.045} = 1028.617 \text{ cir. mils, and}$$

$$d_2 = \sqrt{1028.617} = 32 \text{ mils.}$$

EXAMPLE. — Find the resistance of 500 yards of copper wire 165 mils diameter, the resistance of 1 mile, 230 mils diameter, being 1 ohm. — *Day.*

SOLUTION. — $R_1 = ?$; $R_2 = 1$ ohm,
$l_1 = 500$ yards; $l_2 = 1760$ yards; $d_1 = 165$ mils; $d_2 = 230$ mils.
As before,

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{K_1}{K_1} \times \frac{d_2^2}{d_1^2} = \frac{R_1}{1} = \frac{500}{1760} \times \frac{K_1}{K_1} \times \frac{\overline{230}^2}{\overline{165}^2}.$$

Whence $R_1 = 0.55$ ohm.

For the comparison of the properties of substances, and especially of electrolytes, K, the *specific resistance*, is generally expressed in *microhms per cubic centimeter*, instead of in *ohms per mil-foot.* The microhm is the one-millionth of one ohm.

EXAMPLE. — The specific resistance of copper per cubic centimeter is 1.616 microhms; find the resistance of 10 meters of this wire 2 millimeters in diameter.

SOLUTION. — $R_1 = 1.616$ microhms; $R_2 = ?$

$l_1 = 1$ cm.; $l_2 = 1000$ cm.; $a_1 = 1$ sq. cm.; $a_2 = \frac{\pi}{4} \times (\frac{2}{10})^2$ sq. cm.

$$K_2 = K_1; \ \pi = 3.1416.$$

Using the same formula as before,

$$\frac{1.616}{R_2} = \frac{1}{1000} \times \frac{K_1}{K_1} \times \frac{\pi}{100}.$$

Whence

$$R_2 = \frac{1.616 \times 100 \times 1000}{\pi} = 51{,}438 \text{ microhms} = 0.05 \text{ ohm}.$$

I. Table of Specific Resistances, Specific Gravities, and Specific Heats.

Substances.	Microhms per cu. cm.	Ohms per mil-foot.	Lbs. weight per cu. in.	Grams weight per cu. cm.	Calories of heat per gram.	Temperatures between which sp. heats are given.
						Degrees C.
Steel			0.282	7.8		
Silver, annealed	1.521	9.05	0.379	10.5	0.0568	0–100
Silver, hard drawn	1.652	9.82	0.376	10.4		
Copper, annealed	1.616	9.61	0.320	8.9	0.0933	0–100
Copper, hard drawn	1.652	9.83	0.316	8.78		
Aluminum, annealed,	2.945	17.52	0.093	2.6	0.2122	
Zinc	5.689	33.83	0.253	7.1	0.0935	
Platinum, annealed	9.158	54.47	0.76	21.5	0.0323	0–100
Iron, annealed	9.825	58.44	0.278	7.8	0.1130	0–100
Lead, pressed	19.850	118.05	0.408	11.3	0.0315	0–100
German silver	21.170	125.89	0.307	8.5	0.0946	
Mercury	99.740	572.10	0.490	13.59	0.0333	20–50
2 silver, 1 platinum,	24.660	146.65				
Brass, rolled	5.805	34.54	0.305	8.4	0.09	

From Everett.*	Microhms per cu. cm.	Remarks.
Carre's Carbons at 20° C.	3,927	Carre Carbons give ½ ohm for a cylinder 1 meter long and 1 centimeter in diameter.
Gaudin's Carbons	8,500	
Retort Carbon	67,000	
Graphite Carbon	2,400 to 42,000	

Insulators.	Ohms.	Temp.	Author.
Mica	8.4×10^{13}	20° C.	Ayrton & Perry
Gutta-percha	4.5×10^{14}	24° C.	Latimer Clark
Shellac	9.0×10^{15}	28° C.	Ayrton & Perry
Ebonite	2.8×10^{16}	46° C.	Ayrton & Perry
Paraffin	3.4×10^{16}	46° C.	Ayrton & Perry
Glass	Greater than any of the above.		
Air	Practically infinite.		

* J. D. Everett, *C.G.S. System of Units*, p. 178.

14. **The Relation of Resistance to Weight.**—The weight of a body is obtained by multiplying its length by its cross sectional area, and this product by a constant representing the weight of unit volume, and called the *specific gravity* of the material. For copper the weight per cubic inch is 0.32 lb., and per cubic centimeter 8.9 grams.

For given lengths, the *resistances of conductors vary inversely with their weights.*

EXAMPLE.—Determine the resistance of 100 lbs. of copper wire of a certain length, when another of the same length weighing 500 lbs. has 16.9 ohms resistance.

SOLUTION.— $$\frac{R_1}{R_2} = \frac{W_2}{W_1}.$$

Substituting given values,

$$\frac{R_1}{16.9} = \frac{500}{100}.$$

Whence $$R_1 = \frac{500 \times 16.9}{100} = 84.5 \text{ ohms.}$$

EXAMPLE.—Given that one mile of copper weighing 19.74 lbs. has a resistance of 42.38 ohms, to find the resistance of a mile of copper wire having a diameter of 3 millimeters.

SOLUTION.— $$1 \text{ mile} = 5280 \times 12 = 63{,}360 \text{ in.},$$

$$3 \text{ mm.} = 0.3 \text{ cm.} = 0.3 \times \tfrac{2}{5} = 0.12 \text{ in.},$$

$$W_2 = 63{,}360 \times \overline{0.12}^2 \times 0.7854 \times 0.32 = 229.3 \text{ lbs.}$$

As before, $$\frac{R_1}{R_2} = \frac{W_2}{W_1}.$$

Substituting known terms,

$$\frac{42.38}{R_2} = \frac{229.3}{19.74}.$$

Whence $R_2 = 0.086 \times 42.38 = 3.65$ ohms.

EXAMPLE. — If 1000 feet of copper wire 64 mils in diameter weigh 12.41 lbs., and have a resistance of 2.58 ohms, what will 1 mile of wire 100 mils in diameter weigh, and what will its resistance be?

SOLUTION. — Weight varies as length and cross-section.

Hence $$\frac{W_1}{W_2} = \frac{l_1}{l_2} \times \frac{d_1^2}{d_2^2}.$$

From which $$\frac{W_2}{12.41} = \frac{5280}{1000} \times \frac{\overline{100}^2}{\overline{64}^2} = 12.89.$$

Whence $$W_2 = 12.89 \times 12.41 = 159.96 \text{ lbs.}$$

Also $$\frac{R_2}{R_1} = \frac{l_2}{l_1} \times \frac{d_1^2}{d_2^2} = \frac{5280}{1000} \times \frac{\overline{64}^2}{\overline{100}^2} = 2.16.$$

Whence $$R_2 = 2.16 \times 2.58 = 5.573 \text{ ohms.}$$

EXAMPLE. — Suppose the second wire in the last example were iron instead of copper; what would be its weight and resistance?

SOLUTION. — Weight per cu. in. of iron = 0.278 lb., while that of copper is 0.32 lb. Hence, the total weight in the case of iron would be $159.96 \times \frac{278}{320} = 138.94$ lbs. Specific resistance of iron = 9.825 microhms per cu. cm., while that of copper is 1.616. Hence the total resistance of the iron wire would be $5.573 \times \frac{9.825}{1.616} = 33.88$ ohms.

15. The Relation of Resistance to Temperature. — The resistance of most substances increases with rise of temperature, the most important exception being *carbon*, whose resistance decreases with rise of temperature. The resis-

tance of an incandescent lamp carbon when giving light is only about one-half that when it is cold. The alloy, *manganin*, consisting of 12 per cent manganese, 84 per cent copper, and about 4 per cent nickel, changes very little in resistance with changes of temperature, and is furthermore peculiar in that its resistance increases very slightly up to about 45° C., after which it decreases again.

We express the change in resistance per ohm per degree by calling it the *temperature coefficient* of *resistance.* Thus if R_0 be the resistance at zero C., or any low temperature, a the temperature coefficient of resistance, t the total rise in temperature, then the increase in resistance will be $R_0 \times at$, and the resistance at the higher temperature, R_t, will be expressed by

$$R_t = R_0(1 + at). \tag{16}$$

Example. — Find the resistance of a copper wire at 50° C. when its measured resistance at 15° C. was 10 ohms.

Solution. — $R_0 = R_{15} = 10$ ohms; a for copper $= 0.00406$. (Kennelly & Fessenden); $t = 50° - 15° = 35°$ C. Hence

approx., $R_{50} = 10\ (1 + 0.00406 \times 35) = 11.421$ ohms.

The knowledge of temperature coefficients of resistance is applied practically in the determination of both low and high temperatures. Obviously it is only necessary to measure the resistance of a coil of wire, say platinum, at the required low or high temperature; then knowing the same at any ordinary temperature, and assuming the coefficient to remain constant, the simple application of (16) will obtain the unknown temperature, t.

Example. — Determine the melting temperature of a certain alloy when a coil of platinum wire whose resistance at 0° C. is 10 ohms has a resistance in the melting metal of 50.8 ohms.

Solution. — Applying (16),

$$R_t = R_0(1 + at),$$

and supplying the known terms,

$$50.8 = 10(1 + 0.0034\,t) = 10 + 0.034\,t.$$

From which $t = \dfrac{50.8 - 10}{0.034} = 1200°$ C., approx.

Example. — It is required to determine the temperature of liquefaction of a certain gas when the platinum coil used above has a resistance in the liquid gas of 2.18 ohms, assuming that the temperature coefficient does not alter.

Solution. — Since in this case the temperature becomes lower, the negative sign must be used in the formula; in other words, the resistance at the required temperature is less than at 0°.

Hence $$R_t = R_0(1 - at),$$

and $$2.18 = 10(1 - 0.0034 \times t) = 10 - 0.034\,t.$$

Therefore $t = \dfrac{2.18 - 10}{-0.034} = 230°$ below zero C.

Or, if the positive sign is retained,

$$R_t = R_0(1 + at),$$

and $$2.18 = 10(1 + 0.0034\,t).$$

From which $$t = \frac{2.18 - 10}{+ 0.034} = - 230°,$$

which also means 230 below zero. Either method may be used.

The temperature coefficient of resistance of alloys is in general lower than the coefficients of their component metals. The following are noted:

German silver (60% copper, 25.4% zinc, 14.6% nickel) is,	0.00036
Platinum silver (1 platinum, 2 silver) is	0.00030
Platinoid (German silver and a very little tungsten) is,	0.00022
Ordinary German silver is	0.00044
Carbon has a negative coefficient, possibly about .	— 0.0003
Mercury in glass has a coefficient of	0.0008769
Platinum has a coefficient of	0.0034
Iron has a coefficient of	0.0045
Copper has a coefficient of	0.0042

16. **Conductance and Conductivity.** — It is sometimes convenient, if not necessary, to make use of the *conductance* of a circuit, and the *conductivity* of a material. The conductance of a circuit is the reciprocal of its resistance. The conductivity of a material is the ratio, expressed in per cent, of its conducting power to the conducting power of a standard, often *pure copper*, whose conductivity is called 1, or 100 per cent.

Example. — If the resistance of 1 foot of pure copper wire weighing 1 grain be 0.2106 ohm, and the resistance of a piece of ordinary copper wire 3 feet long weighing 3.45 grains is found to be 0.5782 ohm, how does the conductivity of the latter sample compare with that of pure copper? — *Day.*

SOLUTION. — $$\frac{W_2}{W_1} = \frac{l_2 a_2}{l_1 a_1}.$$

From which $$\frac{a_1}{a_2} = \frac{l_2}{l_1} \times \frac{W_1}{W_2}.$$

Substituting this value for $\frac{a_1}{a_2}$ in the general formula given in (3) section 13, we have,

$$\frac{R_2}{R_1} = \frac{K_2}{K_1} \times \frac{l_2}{l_1} \times \frac{l_2 W_1}{l_1 W_2},$$

assume $K_2 = K_1$, then,

$$\frac{R_2}{R_1} = \frac{l_2^2}{l_1^2} \times \frac{W_1}{W_2}.$$

From which

$$R_2 = R_1 \frac{l_2^2}{l_1^2} \times \frac{W_1}{W_2} = 0.2106 \times \left(\frac{3}{1}\right)^2 \times \frac{1}{3.45} = 0.5494 \text{ ohm.}$$

This would be the resistance of the second wire if it were pure copper. But its actual resistance is 0.5782 ohm. Therefore its conductivity is

$$\frac{5494}{5782} = 95 \text{ per cent.}$$

EXAMPLE. — A circuit consists of a battery whose resistance is 2 ohms in series with two resistances of 10 ohms and 15 ohms respectively. Find the conductance of the circuit.

SOLUTION. — Since all parts are in series the total resistance is the sum of all parts.

Hence $$R = 2 + 10 + 15 = 27 \text{ ohms.}$$

Therefore conductance $c = \frac{1}{27} = 0.037$.

17. Compound Circuits. — A compound circuit is one with two or more branches, or *shunts*. Thus in Fig. 1 the

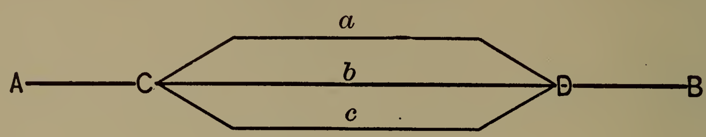


Fig. 1.

branches *a*, *b*, and *c* are called *shunts* relative to each other. In other words, they are connected in *parallel* with each other at the junction points *C* and *D*.

EXAMPLE. — Obtain the formula for the combined resistance of *a* and *b*. Also for *a*, *b*, and *c*.

SOLUTION. — Let E be the E.M.F. between C and D, and R_1, R_2, and R_3, the resistances of *a*, *b*, and *c*, respectively. The current flowing in the branches will then be

$$I_1 = \frac{E}{R_1}, \quad I_2 = \frac{E}{R_2}, \quad \text{and } I_3 = \frac{E}{R_3}.$$

The total current

$$I = I_1 + I_2 + I_3 = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right).$$

But $I = \frac{E}{R}$ where R is the combined resistance.

Therefore $$\frac{E}{R} = E\left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2}\right).$$

But $\frac{1}{R}$ is the total conductance of the parallel paths, and

$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ is the sum of the respective conductances,

and $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

Hence, *the total conductance of parallel circuits is the sum of the individual conductances.* Reducing to a common denominator, equating the numerators, and solving for R, we get

$$R_1 R_2 = RR_2 + RR_1 \text{ for the first and second.}$$

Then $$R(R_1 + R_2) = R_1 R_2.$$

From which $$R = \frac{R_1 R_2}{R_1 + R_2}. \qquad (17)$$

Hence *the resistance of two circuits in parallel, or shunts, is equal to the product of the two separate resistances divided by their sum.*

Also for all three paths,

$$R_1 R_2 R_3 = RR_2 R_3 + RR_1 R_2 + RR_1 R_3.$$

Whence $$R(R_1 R_2 + R_1 R_3 + R_2 R_3) = R_1 R_2 R_3.$$

Therefore $$R = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}. \qquad (18)$$

Or in words, *the combined resistance is equal to the product of the separate resistances divided by the sum of their products taken two at a time.*

In (18), if the resistances are equal to each other $R = \frac{R_1^3}{3 R_1^2} = \frac{R_1}{3}$, or one-third of one resistance. Making this rule general, if R_1 represent each resistance and n the number of such in parallel, then the combined resistance will be

$$R = \frac{R_1}{n}. \qquad (19)$$

Example. — Find the resistance of 10 incandescent lamps in parallel, when the resistance of each, hot, is 200 ohms.

Solution. — Using (19), $R = \frac{200}{10} = 20$ ohms.

Example. — Calculate the resistance of 3 wires in parallel, the resistances being, respectively, 5, 10, and 15 ohms.

Solution. — Applying (18),

$$R = \frac{5 \times 10 \times 15}{5 \times 10 + 5 \times 15 + 10 \times 15} = 2.72 \text{ ohms.}$$

18. Current Intensity in Compound Circuits. — Example. — A current of 42 amperes flows in a circuit composed of 3 branches of 5, 10, and 20 ohms resistance, respectively. Find the current intensity in each branch.

Solution. — From Ohm's law currents are manifestly proportional inversely to resistances, and directly to conductances. The conductances of the branches are, respectively, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, total $\frac{7}{20}$, through which the total current of 42 amperes flows. Therefore,

$$\tfrac{7}{20} : 42 :: \tfrac{1}{5} : x_1; \text{ whence, } x_1 = 24 \text{ amperes.}$$
$$\tfrac{7}{20} : 42 :: \tfrac{1}{10} : x_2; \text{ whence, } x_2 = 12 \text{ amperes.}$$
$$\tfrac{7}{20} : 42 :: \tfrac{1}{20} : x_3; \text{ whence, } x_3 = 6 \text{ amperes.}$$

19. Resistance and Drop of Potential. — According to Ohm's law, $E = IR$, it is clear that potential difference, or electromotive force, is directly proportional to the resistance, the current remaining the same; or, we obtain *drop*, or *difference* of *potential*, between two points by multiplying the resistance between the points by the current flowing in the circuit.

EXAMPLE. — A cell of battery is connected in series with the following resistances: a 5 ohm coil of wire, a 10 ohm coil, and 3 coils in parallel whose individual resistances are 5, 10, and 20 ohms, respectively. Make a diagram of the circuit, and find the drop of potential over each of the resistances, and also over the cell whose constants are 2 volts of E.M.F., and $\frac{1}{2}$ ohm internal resistance.

SOLUTION. — The total E.M.F. of 2 volts is used up, so to speak, in the total resistance. In other words, 2 volts is the total drop over the total resistance in circuit. The proportion of the drop over each resistance is therefore the same as the proportion of the resistances to the whole resistance.

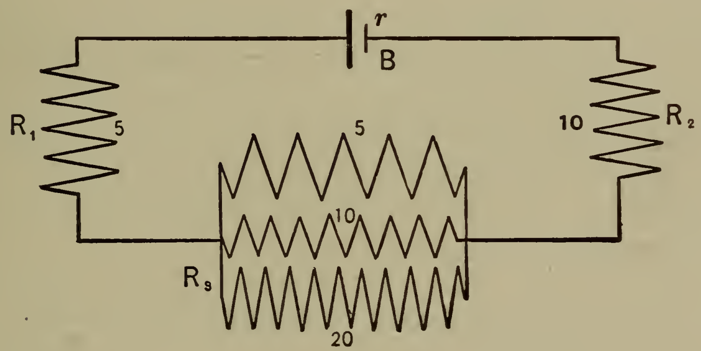


Fig. 2.

The total resistance is

$$R = r + R_1 + R_2 + R_3.$$

Substituting,

$$R = 0.5 + 5 + 10 + \frac{5 \times 10 \times 20}{5 \times 10 + 5 \times 20 + 10 \times 20} = 18.36 \text{ ohms.}$$

Therefore,

Ohms.	Volts.	Ohms.	Volts.	
18.36	: 2 ::	5	: x_1;	whence, $x_1 = 0.544$ volts.
18.36	: 2 ::	10	: x_2;	whence, $x_2 = 1.088$ volts.
18.36	: 2 ::	0.5	: x ;	whence, $x = 0.056$ volts.
18.36	: 2 ::	2.86	: x_3;	whence, $x_3 = 0.312$ volts.

A voltmeter applied to a cell when no circuit is connected to it measures the total E.M.F. of the cell. But if a circuit is at the same time attached to the cell the voltmeter measures the total drop over the resistance of the circuit. The difference between these two readings of the voltmeter gives the drop over the internal resistance of the cell under the given conditions. The same thing is true of the dynamo.

Example. — What is the resistance of the battery whose total E.M.F. is $9\frac{1}{2}$ volts, if, when it is connected in series with three resistances of 6, 3, and 9 ohms, a voltmeter connected across the 6 ohm coil reads 3 volts?

Solution. — Making the proportions as above,

Ohms.	Volts.	Ohms.	Volts.	
6 :	3 ::	9 :	x_1;	whence, $x_1 = 4\frac{1}{2}$ volts.
6 :	3 ::	3 :	x_2;	whence, $x_2 = 1\frac{1}{2}$ volts.

Total external drop $= 3 + 4\frac{1}{2} + 1\frac{1}{2} = 9$ volts. Therefore the internal drop $= 0.5$ volt. Hence,

$6 : 3 :: x : 0.5$; whence, $x = 1$ ohm, the internal resistance of the battery.

20. **Original Problems.** — 1. A Bunsen cell has the specific resistance of its inner and outer portions made alike, each being 9 ohms per cubic centimeter. The

central electrode is a $\frac{1}{2}$ inch cylinder of electric light carbon. The outer element is a plate of zinc 6 inches wide bent into a hollow cylinder. What will be the resistance of the cell when the jar is filled 4 inches deep with the solutions? $r = 0.165$ ohm.

2. How long is a 68 mil wire whose resistance is 40 ohms, when 2 miles of the same kind of wire 100 mils diameter have a resistance of 11 ohms?

$l_1 = 3.363$ miles.

3. There are two conductors, one of 35 ohms resistance, 1728 feet long, and 12 square millimeters cross sectional area, specific resistance, 7; the other, 14 ohms resistance, 432 feet long, and 8 square millimeters cross section. What is the specific resistance of the latter? — *Sloane.*

$K_2 = 7.4$ ohms.

4. What must be the length in feet and diameter in mils of a German silver wire, specific resistance 125 ohms, so that it may have a total resistance of 1250 ohms, when 500 feet of the same material of twice the diameter have a resistance of 156.25 ohms? $l_1 = 1000$ feet.

$d_1 = 10$ mils.

5. Find the resistance of 15 miles of iron wire 0.3 inch in diameter, having given that the resistance of one mil-foot of iron wire is 59.1 ohms. — *Day.*

$R_2 = 52$ ohms.

6. Find the specific resistance in microhms per cubic centimeter of a wire, having given that its specific resistance per mil-foot is 9.8 ohms.

$$K = \frac{9.8\,(0.001 \times \frac{5}{2})^2 \times \frac{1}{4}\pi \times 10^6}{12 \times \frac{5}{2}} = 1.603 \text{ microhms.}$$

7. Find the resistance of $\frac{1}{2}$ mile of aluminum wire 0.115 inch diameter, specific resistance per mil-foot being 17.52 ohms. Also find the specific resistance in microhms per cubic centimeter, and compare with table 1.

$R = 3.49$ ohms.
$K = 2.866$ mich. Table gives 2.945.

8. If the resistance of a wire 3 meters long and weighing 3 grams is 5.88 ohms, what is the specific resistance in microhms per cubic centimeter, its specific gravity being 20.337? — *Day.*

SOLUTION. —

$$a_1 \times 300 \times 20.337 = 3 \text{ grams.}$$

Therefore

$$a_1 = \frac{3}{300 \times 20.337} \text{ sq. cm.}$$

$$R_1 = K_1 \frac{l_1}{d_1^2}, \text{ and } R_2 = K_2 \frac{l_2}{d_2^2}.$$

Whence

$$R_2 = R_1 \times \frac{l_2}{l_1} \times \frac{d_1^2}{d_2^2} = 5.88 \times \frac{1}{300} \times \frac{3}{300 \times 20.337}$$

$$= 9.638 \text{ microhms.}$$

Since $K_1 = K_2$, and $l_2 = 1$.

9. What is the resistance of a wire 3 feet long weighing 5 grains, when it is known that one of the same material 10 feet long weighs 16 grains and has a resistance of 0.156?

SOLUTION. —

$$R_2 = R_1 \left(\frac{l_2}{l_1}\right)^2 \times \frac{W_1}{W_2} = 0.156 \times \left(\frac{3}{10}\right)^2 \times \frac{16}{5} = 0.045 \text{ ohms.}$$

10. How many pounds of a certain size of copper wire must I purchase to have a length of 500 feet and a resistance of 0.818 ohm, when it is already known that 20 pounds require 1012 feet and have a resistance of 1.63 ohms?

$W_2 = 9.7$ lbs.

11. What is the temperature coefficient of resistance of a certain metal when it is found that a coil of wire made of it and having 10 ohms resistance at 15° C., increases to 16.8 ohms when heated to 215° C.?

$a = 0.0034$.

12. A standard resistance of German silver wire is marked "100 ohms, right at 15° C." What is the correction necessary when the resistance is used at 28° C.?

$R_{28} = 100.572$ ohms.
Correction, 0.00572 per ohm.

13. It is found by experiment that a spool of platinoid wire at 25° C. has a resistance of 50.275 ohms. How should it be marked for a standard correct at 0° C.?

$R^{\circ} = 50$ ohms.

14. Find the fusion temperature of a certain metal when a 50 ohm coil of platinum wire "right at 0° C." has a resistance of 220 ohms in the molten metal.

$t = 1000°$ C.

15. Two samples of copper wire were brought to be tested for conductivity, so a length of 20 feet was cut from each sample. The first weighed 150 grains and had a resistance of 0.613 ohm, the second weighed 164 grains and had a resistance of 0.547 ohm. Find the

conductivity of each sample. Pure copper has a resistance of 0.2106 ohm and weighs 1 grain for 1 foot in length. —*Day.* $C_1 = 91.6\%$.
$C_2 = 93.9\%$.

16. The resistance of 1 mile of copper wire whose diameter was 0.065 inch was found to be 15.73 ohms. The resistance of a wire of pure copper 1 foot long and 0.001 inch in diameter is 9.94 ohms. Find the conductivity of the first wire. — *Day.* $C_1 = 78.96\%$.

17. Find the total conductance of a circuit in which are a battery whose resistance is $1\frac{1}{2}$ ohms, a wire resistance of $3\frac{1}{2}$ ohms, another group of 2 wires in parallel whose separate resistances are 5 ohms and 15 ohms respectively. Also, how many amperes will flow, the battery E.M.F. being 2 volts? $c = 0.114$
$I = 0.228$ ampere.

18. How much current will flow through each of the two parallel resistances in 17?
$I_1 = 0.057$ amp. in 15-ohm wire.
$I_2 = 0.171$ amp. in 5-ohm wire.

19. Find the drop of potential over the parallel portion of 17. $E = 0.855$ volt.

20. A dynamo machine whose resistance is 0.25 ohm is connected by lead wires whose resistance is 0.25 ohm to 40 incandescent lamps in parallel, each having a resistance of 220 ohms hot. How many volts E.M.F. must the dynamo generate to give each lamp $\frac{1}{2}$ ampere of current? $E = 120$ volts.

21. When 4 wires of 2.5, 5, 10, and 20 ohms resistance are joined in parallel, how much current will flow in each when a battery whose E.M.F. is 19 volts and whose internal resistance is 5 ohms is connected to them? $I = 3$ amperes.

$$\left.\begin{aligned} I_1 &= 1.6 \\ I_2 &= 0.8 \\ I_3 &= 0.4 \\ I_4 &= 0.2 \end{aligned}\right\} \text{amperes.}$$

22. How many incandescent lamps, each requiring 1 ampere and having a resistance of 50 ohms, can be put in parallel, or multiple, on a machine giving 60 volts and having an internal resistance of 0.02 ohm, when the lead wires have a resistance of 0.1 ohm? $n = 83$ in parallel.

23. A dynamo giving 580 volts, having a resistance of 1 ohm, is connected to a circuit containing in parallel 80 groups of lamps, each group having 5 lamps in series requiring $\frac{1}{2}$ ampere of current. Find the voltage between the lines at the lamps, and also of each lamp. Also find the resistance of the wires the drop of potential over which is 40 volts, and find the percentage of the dynamo power used in the lamps, and the percentage lost in the machine itself. $E = 500$ volts for each group.

$e = 100$ volts for each lamp.

$R = 1$ ohm in line.

Lamps get 86.2% of total power.

Internal loss 6.9% of total power.

Line loss 6.9% of total power.

24. A dynamo machine when not connected to any circuit, but running under excitation at full speed, gives a

pressure at the brushes of 120 volts, as measured by a Weston voltmeter. When the full load of 60 lamps is put on the machine, the voltmeter reads 113.8 volts while the ammeter in the circuit reads 32 amperes. The voltmeter is now taken to the center of the system of lamps where it reads 109.5 volts. Find the drop on the line and in the machine. Also find the resistance of the line, of the lamps (average), and of the machine. Also determine the ratio of external to total energy of the machine.

Internal drop = 6.2 volts.
Line drop = 4.3 volts.
Total drop = 10.5 volts.
Lamp E = 109.5 volts.
Resistance of machine = 0.194 ohm.
Resistance of line = 0.134 ohm.
Resistance of lamps = 205.2 (average).
Ratio of ex. to total energy = 94.8%.

25. Device M requires 5 amperes at 110 volts. A 220 volt circuit is available, and a 20 ohm rheostat capable of

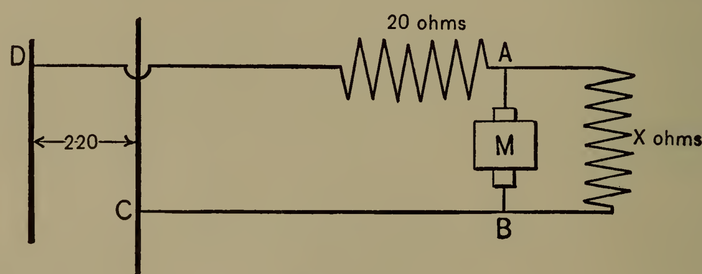


Fig. 3.

carrying 5 amperes is at hand to put in series with the device. Now what resistance as a shunt to the device will enable it to operate under normal current?

Since between A and B only 110 volts are permissible, there must be a drop in the 20 ohms of

$$220 - 110 = 110 \text{ volts.}$$

Therefore the resistance in the circuit between A and B is given by the proportion,

$$110 \text{ volts} : 20 \text{ ohms} :: 110 \text{ volts} : x; \text{ whence } x = 20 \text{ ohms.}$$

But this is the combined resistance of circuit M and shunt, and by (17)

$$R = \frac{R_1 R_2}{R_1 + R_2}.$$

Let R_1 be the resistance of circuit $M = \frac{110}{5} = 22$ ohms. Therefore, substituting values,

$$20 = \frac{22\,R_2}{22 + R_2};$$

From which

$$R_2 = 220 \text{ ohms} = \text{shunt resistance.}$$

The shunt will carry $\frac{220}{40} - 5 = 0.5$ ampere.

The same result may be accomplished by putting additional resistance in series. In this case, since the total current is 5 amperes, the total resistance must be

$$220 \div 5 = 44 \text{ ohms.}$$

This will require $44 - 20 = 24$ ohms additional resistance in series capable of carrying 5 amperes, and is much better.

26. Suppose a 500 volt street car line is called upon to supply stationary motors in a factory requiring only 220 volts and 10 amperes of current. Suppose further that a regulator rheostat of 18 ohms resistance can be put in

series with the motors. What resistance now must be in parallel with the motors? Also what additional series resistance would cause the motors to work normally?

Shunt resistance = $39\frac{3}{5}$ ohms.
Series resistance = 10 ohms.

The latter would be used in practice, if either.

IV.

ELECTRICAL ENERGY.

21. Electrical and Mechanical Energy. — Energy, or the capacity for doing work, is measured by the same units as work. In the C.G.S. system, the unit is the *erg*, or the work of 1 dyne of force acting through 1 centimeter of space. In ordinary mechanical work, the unit is the foot-pound, or the work of 1 pound of force acting through 1 foot of space. In all cases work and energy are expressed by the product of force and distance. For estimating *activity*, or *power*, that is, the rate of doing work, the *horse-power* equal to 33,000 foot-pounds per minute is used as the unit. For some purposes also the kilogram-meter is used as the unit of mechanical energy, being the work of 1 K. acting through 1 m. of space. As a smaller unit the *gram-centimeter* is chosen.

As the unit of electrical power the *watt* is used, and for larger values the *kilowatt* is often convenient. The capacity of dynamos is always expressed in the latter unit.

Expressing electrical power in watts by P, the formula is

$$P = EI. \tag{20}$$

Making the substitutions for E and I according to the law of Ohm,

$$P = I^2R, \text{ and } P = \frac{E^2}{R}.$$

So that the formula for P can be used which is most convenient.

It is sometimes necessary to make use of the whole amount of electrical work in a certain time. Representing the time in seconds by t, and the *joules*, or *volt-coulombs*, by W, the rule for the whole number is,

$$W = EIt. \tag{21}$$

EXAMPLE. — Reduce to ergs the energy employed per second by 110 volts of E.M.F. sending 10 amperes of current through a circuit.

SOLUTION. —

1 volt $= 10^8$ C.G.S. units; 1 ampere $= 10^{-1}$.

Therefore 1 watt $= 10^8 \times 10^{-1} = 10^7$ ergs per second.

From (20),

$$P = EI = 110 \times 10 = 1100 \text{ watts.}$$

$$1100 \text{ watts} \times 10^7 = 11 \times 10^9 \text{ ergs per second.}$$

EXAMPLE. — How many ergs of work will be done by 110 volts and 5 amperes in half an hour?

SOLUTION. —

$$W = EIt = 110 \times 5 \times 1800 \times 10^7 = 99 \times 10^{11} \text{ ergs.}$$

EXAMPLE. — How many watts of electrical energy are equivalent to 1 H.P. of mechanical energy.

SOLUTION. —

1 watt $= 10^7$ ergs; 1 H.P. $= 550$ ft.-lbs. per second.

1 m. $= 3.28$ feet.; 1 K. $= 2.2$ lbs.; 1 K.-m. $= 3.28 \times 2.2$ $= 7.23$ ft.-lbs.

Therefore

$$1 \text{ H.P.} = \frac{550}{7.23} = 76.072 \text{ K.-m. per second.}$$
$$= 76.072 \times 1000 \times 100$$
$$= 76.072 \times 10^5 \text{ gr.-cm. per second.}$$

The acceleration of gravity is approximately 980 cms. per second, which is the ratio of the gravitation or practical system of units to the C.G.S. system. Hence the value given above is in C.G.S. units.

$$76.072 \times 10^5 \times 980 = 745.5056 \times 10^7 \text{ dyne-cm., or ergs.}$$

Therefore in round numbers

$$746 \text{ watts} = 1 \text{ H.P.} \qquad (22)$$

22. **Electrical and Heat Energy.** — No transformation of energy can take place without more or less waste in the form of heat, or unavailable energy. In every process of producing electrical energy by transformation from mechanical or chemical energy, a considerable portion is lost as heat in the generating device. Its transmission to the points of utilization is also attended by much loss in heating the conductors. Again, whether motors are used for power, or lamps for lighting, there is a still further transformation into heat; in fact, being necessary in the latter in order to produce light. Hence it becomes very important to understand the exact relations of electricity to heat energy.

It has already been shown that electrical power is represented by I^2R, $\frac{E^2}{R}$ and EI. The heat into which this power may be transformed is therefore directly porportional to these quantities.

EXAMPLE. — Compare the heat produced in the conductors in transmitting 50 amperes through a resistance of 10 ohms with that when 20 amperes are sent through the same circuit.

SOLUTION. — $H_1 \propto I_1^2 R_1$,

and $H_2 \propto I_2^2 R_2$. But $R_1 = R_2$.

Hence $H_1 \propto 50^2 \times 10 = 25{,}000$;

and $H_2 \propto 20^2 \times 10 = 4000$.

Therefore $$\frac{H_1}{H_2} = \frac{25{,}000}{4000} = 6\tfrac{1}{4}.$$

That is while the current in the first case is only $2\frac{1}{2}$ times that in the second, the heat produced is $6\frac{1}{4}$ times as great. Therefore, *the resistance remaining unchanged, the heat varies as the square of the current.*

EXAMPLE. — How will the line loss in carrying 10 amperes through a 5 ohm circuit compare with that when the circuit has only 2 ohms resistance?

SOLUTION. — $H_1 \propto I_1^2 R_1$, and $H_2 \propto I_2^2 R_2$. But $I_1 = I_2$.

Hence $$\frac{H_1}{H_2} = \frac{10^2 \times 5}{10^2 \times 2} = 2\tfrac{1}{2}.$$

That is, *Heat varies directly as the resistance when the current is the same.*

EXAMPLE. — The voltage of the dynamo is 116, of the lamps 110, the line resistance is 10 ohms. How will the line loss compare with the loss if the resistance were only 5 ohms, other conditions unchanged?

SOLUTION. — Line drop $= 116 - 110 = 6$ volts. Then

$$\frac{H_1}{H_2} = \frac{\dfrac{E_1^2}{R_1}}{\dfrac{E_2^2}{R_2}} = \frac{\dfrac{6^2}{10}}{\dfrac{6^2}{5}} = \frac{1}{2}.$$

Example. — When the line drop is 5 volts and the current 10 amperes, how much more heat is produced in transmission than in a circuit carrying 10 amperes, and having a drop of 2 volts?

Solution. —

$$\frac{H_1}{H_2} = \frac{I_1E_1}{I_2E_2} = \frac{10 \times 5}{10 \times 2} = 2\tfrac{1}{2} \text{ times.}$$

To measure absolutely the amount of heat certain units and constants are necessary. Joule* first found by experiment that 772† foot-pounds of mechanical energy will raise 1 *pound of water* 1° *Fahrenheit*, which he chose to call a unit of heat; 772 is then "Joule's Equivalent," the amount of mechanical work equivalent to 1 unit of heat. This would be 1390.59 ft.-lbs. per degree C., or 423.85 kilogram-meters per kilogram-degree of heat. Later experiments made in the U. S. give the value of the equivalent as 427.52 K.-m. If the *gram-degree* of water is used as the unit, 427.52 gram-meters are equivalent to the small calorie. Representing the mechanical equivalent by J and reducing to ergs,

$$J = 427.52 \times 100 \times 980 = 4.19 \times 10^7 \text{ ergs.}$$

1 calorie (water-gram-degree) $= 4.19 \times 10^7$ ergs of work.

Example. — To find the ratio between joules of electrical energy and calories of heat energy.

* British Association, 1843; also Phil. Mag., Vol. XXXII., 1843. See *The Second Law of Thermodynamics* in Harpers' "Scientific Memoirs," p. 111.

† More recent researches fix this at 778 foot-pounds.

SOLUTION. —

$$\text{Ergs} = \text{watts} \times 10^7 = IE \times 10^7 = 10^7 \text{ ergs in 1 watt.}$$

$$1 \text{ calorie} = 4.19 \times 10^7 \text{ ergs.}$$

The ratio desired is then

$$\frac{\text{watt}}{\text{calorie}} = \frac{10^7}{4.19 \times 10^7} = \frac{1}{4.19} = 0.24.$$

Therefore, since the *number* of units is inversely as the *size* of the unit the number of calories corresponding to any number of watts, IE, will be

$$H \text{(calories)} = IE \times 0.24. \qquad (23)$$

To obtain calories multiply watts or joules by 0.24.

The *specific heat* of a substance is the amount of heat expressed in calories necessary to change the temperature of 1 gram of it 1° C. Water is the standard and has a specific heat of one.

EXAMPLE. — Suppose no heat lost by radiation, how hot will a copper conductor 1000 feet long and 0.25 inch diameter and carrying 1000 amperes become in one second? In 10 minutes?

SOLUTION. — $R = K\dfrac{l}{d^2} = 9.8 \dfrac{1000}{250^2} = 0.1568$ ohm.

The volume $= 1000 \times 12 \times \overline{0.25}^2 \times 0.7854 = 589.04$ cu. in.

The weight $= 589.04 \times 145.45$ (wt. per cu. in.) $= 85{,}675.9$ grams.

$$H = \overline{1000}^2 \times 0.1568 \times 0.24$$
$$= 37{,}632 \text{ calories per second.}$$

Heat acquired by the wire = mass × sp. heat × t (rise).

$$H = 85{,}675.9 \times 0.0933 \times t = 37{,}632.$$

Therefore $t = \dfrac{37{,}632}{85{,}675.9 \times 0.0933} = 4.7^\circ$ above air.

In 10 min. this would be

$$4.7^\circ \times 600 = 2820^\circ \text{ if possible.}$$

Copper melts at 1050° C.

Although it is not known definitely because of so many influencing conditions, yet approximately $\frac{1}{4000}$ *of a heat unit is radiated from each square centimeter of external surface of a conductor for each degree rise of temperature above the air.*

EXAMPLE. — How hot will the conductor in the last problem become, allowing the above rate of radiation to take place?

SOLUTION. —

$$\text{Surface } s \text{ of wire} = (1000 \times 12 \times \tfrac{5}{2})(0.25 \times \tfrac{5}{2}) \times \pi$$
$$= 58{,}905 \text{ sq. cm.}$$

Now the temperature will continue to rise until the rate of radiation is equal to the rate of development of heat, when it will remain stationary.

$$\text{Rate of production of heat} = I^2R \times 0.24 = \overline{1000}^2 \times 0.157 \times 0.24 = 37{,}680 \text{ calories per second.}$$

$$\text{Rate of radiation of heat} = \tfrac{1}{4000} \times 58{,}905 \times t,$$

in which t is the rise of temperature.

Therefore,

$$58{,}905 \times t \times \tfrac{1}{4000} = 37{,}680 \text{ calories per second.}$$

Therefore $t = \dfrac{37{,}680 \times 4000}{58{,}905} = 2552^\circ$ C.

This is sufficient to melt the wire.

EXAMPLE. — Making use of the principles already given, find the diameter of a piece of copper fuse wire for the protection of a street-car motor, in order to carry 40 amperes maximum current, the specific resistance of copper per cubic centimeter being 1.616 microhms, and taking its fusing point at 1000° C.

SOLUTION. — Represent the diameter of the wire in centimeters by d, and take 1 centimeter as the length for purposes of the calculation, since the length does not materially affect the point of fusion. The resistance of this will be

$$R = \frac{1.616 \times 1}{10^6 \times \left(d^2 \times \frac{\pi}{4}\right)} = \frac{1.616 \times 4}{10^6 \times \pi d^2} \text{ ohms.}$$

From (23)

$$H = I^2R \times 0.24 = 40^2 \times \frac{1.616 \times 4}{10^6 \times \pi d^2} \times 0.24 \text{ cal.}$$

This is the heat developed per second in the fuse-wire. For the purposes of protection, the rate of radiation must not become equal to this till the temperature of fusion is reached, or 1000° C. The heat radiated per second at 1000° C. is, for 1 cm. length,

$$H = \frac{1}{4000} \times \pi d \times 1000^\circ.$$

Therefore

$$\frac{1}{4000} \times \pi d \times 1000 = \overline{40}^2 \times \frac{1.616 \times 4}{10^6 \times \pi d^2} \times 0.24.$$

From which

$$d^3 = \overline{40}^2 \times \frac{1.616 \times 4}{10^6 \times \pi^2} \times 0.24 \times \frac{4000}{1000}.$$

Hence

$$d^3 = \frac{\overline{40}^2 \times 1.616 \times 0.000391}{1000} = 0.0010196.$$

Therefore

$$d = \sqrt[3]{0.0010196} = 0.1 \text{ cm.} = 40 \text{ mils} = \text{No. } 18.$$

It will be clear that the final expression for d^3 just above may be written so as to make a general rule; for $\overline{40}^2 = I^2$, $1.616 = K$, and $1000 = t$. Therefore the formula for general application is

$$d^3 = \frac{I^2 \times K \times 0.000391}{t^\circ}.^* \qquad (24)$$

From the foregoing principles tables of "safe carrying capacity" are worked out. Some allow higher temperatures as "safe" than others. From experiment and calculation Dr. A. E. Kennelly has arranged a table of carrying capacities, now generally accepted, which are about 20 per cent lower than those of the National Code.

It can also be seen from what precedes that the heat increases with I^2 and with $\frac{1}{d^3}\cdot$ Hence

$$H \propto \frac{I^2}{d^3}.$$

Now 30° C. above the surrounding air is at least a safe limit, though some would allow higher temperatures. In order that this temperature be not exceeded, the following empirical formula may be used for finding I, the safe current for a wire of a given diameter d;

$$I = \frac{\sqrt{d^3}}{2500}. \qquad (25)$$

* A convenient practical formula for resistance due to any temperature t is

$$R_t = \frac{9.6\,(1 + .004\,t)\,l}{d^2}.$$

23. **Efficiency of Transformation and Transmission of Energy.** — The efficiency of conversion of an electrical generator is the ratio of the total electrical energy generated to the mechanical energy supplied to the machine. Representing the efficiency of conversion by C,

$$C = \frac{\text{E.H.P. (total)}}{\text{M.H.P.}}. \tag{26}$$

The *commercial efficiency* of a generator is the ratio of the electrical energy supplied to the external circuit to the mechanical energy used in driving the machine. Therefore

$$\text{Com. Eff.} = \frac{\text{E.H.P. (external)}}{\text{M.H.P.}}. \tag{27}$$

The *electrical efficiency* of a generator is the ratio of the external electrical energy to the total electrical energy developed in the armature of the machine. Hence,

$$\text{Elec. Eff.} = \frac{\text{E.H.P. (external)}}{\text{E.H.P. (total)}}. \tag{28}$$

Station efficiency is often employed to designate the ratio of the power delivered to the line terminals on the switchboard to the power supplied to the engine by the steam. The difference between these two quantities is the total loss in the station beginning at the engine.

Plant efficiency may also be used to represent the ratio of the energy delivered to the lamp or motor terminals to the energy supplied to the steam engine. The latter is important in judging of the performance of the whole system and in fixing charges to consumers.

The various losses in a plant are friction and radiation losses in the engine ; losses in belting and so forth ; fric-

tion, hysteresis, eddy current, and I^2R losses in dynamos, and I^2R losses in the switchboard connections and in the line. Evidently that plant will be most efficient which has observed the means of reducing each of these losses to the lowest possible.

Example. — What is the commercial efficiency of a dynamo, when a dynamometer shows that it absorbs 8 H.P. of mechanical energy while furnishing 92 incandescent lamps with 46 amperes of current at 115 volts?

Solution. — $\text{E.H.P.} = \dfrac{115 \times 46}{746} = 7.$

$$\text{Com. Eff.} = \frac{7}{8} = 87.5\%.$$

Example. — The resistance of an armature is 0.02 ohm and the shunt fields have a resistance of 25 ohms. The generator absorbs 10 H.P. and delivers 50 amperes to the line, while the brush E.M.F. is 118.8 volts. What is the efficiency of conversion, commercial efficiency, and the electrical efficiency of the machine? Also the per cent of loss in field magnets and armature?

Solution. — External watts $= 118.8 \times 50 = 5940.$

$$\text{Field current} = \frac{118.8}{25} = 4.75 \text{ amperes;}$$

$$\text{Field watts} = 118.8 \times 4.75 = 564.3.$$

$$\text{Armature watts loss} = \overline{54.75}^2 \times 0.02 = 60.$$

Therefore

$$\text{Total watts} = 5940 + 564.3 + 60 = 6564.3.$$

$$10 \text{ H.P.} = 7460 \text{ watts supplied.}$$

Hence

Eff. Con., $C = 6564.3 \div 7460 = 87.99\%$.
Com. Eff. $= 5940 \div 7460 = 79.62\%$.
Elec. Eff. $= 5940 \div 6564.3 = 90.48\%$.
Total machine loss $= 60 + 564.3 = 624.3$ watts.
Electric loss $= 624.3 \div 6564.3 = 9.51\%$.

These figures would show a very badly designed machine for modern times.

EXAMPLE. — Calculate the efficiency of a long distance line when 50 amperes at 3600 volts are supplied to it, its resistance being 10.8 ohms.

SOLUTION. — Watts supplied $= 3600 \times 50 = 180{,}000$.
Line loss $= \overline{50}^2 \times 10.8 = 27{,}000$ watts.

Therefore,

Watts delivered $= 180{,}000 - 27{,}000 = 153{,}000$.

Hence Line eff. $= \frac{153000}{180000} = 85\%$,

and Line loss $= \frac{27000}{180000} = 15\%$.

EXAMPLE. — A plant consists of a generator of 94% efficiency, step-up transformer (lower to higher voltage) 96% efficiency, line 90% efficiency, lowering or step-down transformer 95%. The engine that drives the dynamo has an efficiency of 88%. If 339.32 K.W. of energy is delivered at secondaries of step-down transformers, calculate the plant efficiency and the H.P. supplied to the engine; also the station efficiency.

SOLUTION. — Let 100% = energy supplied to engine, $0.88 \times 0.94 \times 0.96 \times 0.90 \times 0.95 = 67.89\%$ = plant efficiency.

Therefore

$$67.89\% = 339.32 \text{ K.W.}$$
$$1\% = 5.$$
$$100\% = 500 \text{ K.W. supplied to engine.}$$
$$\text{H.P. of engine} = \frac{500000}{746} = 670.$$

The station efficiency = 88% of 94% = 82.72%, if we include the belt losses and slipping losses in the 12% engine loss.

The efficiency of most machinery is much higher at full load than at very small loads. This is largely true of engines, transformers, dynamos, and motors. However, in the latter the difference between full-load efficiency and say one-half load is small. The efficiencies as previously given are for full load. In engines the losses are almost constant for all loads. Hence an engine which at full load would have an efficiency, say of 90%, at half load would have only about 82% efficiency. To illustrate, take 100 H.P. supplied, 90 H.P. delivered, making 10 H.P. lost. Half load delivers 45 H.P., requiring $45 + 10 = 55$ to be supplied. Efficiency $= \frac{45}{55} = 82\%$. Therefore all engines should keep loaded as fully as possible. For dynamos, motors, and large transformers, the efficiency changes but little with change of load, if the load is not allowed to fall below about one-half.

Example. — A street car with its load weighs 10 tons, and the average grade of the street is 5%. Allowing an efficiency for the motors and gearing of 75%, and a horizontal speed of 10 miles an hour, what power must be applied to the motor?

SOLUTION. — The horizontal effort may be taken as 25 lbs. per ton, making $10 \times 25 = 250$ lbs. for this car. Work done horizontally per minute is then

$$W_1 = \frac{5280 \times 10 \times 250}{60} = 220{,}000 \text{ ft.-lbs.}$$

The vertical lift per minute is

$$\frac{5\% \text{ of } 5280 \times 10}{60} = 44 \text{ ft. per min.}$$

Work done vertically per minute is therefore

$$W_2 = 10 \times 2000 \times 44 = 880{,}000 \text{ ft.-lbs.}$$

Total work required at the wheels of car

$$= 220{,}000 + 880{,}000 = 1{,}100{,}000 \text{ ft.-lbs.}$$

Horse-power necessary to be taken from the line is

$$\frac{1{,}100{,}000}{0.75} \div 33{,}000 = 44.4 \text{ H.P.}$$

EXAMPLE. — What efficiency will be necessary in a stationary motor in order that it may develop 450 volts counter E.M.F. when the applied E.M.F. is 500 volts?

SOLUTION. — Since the output is the product of the current and the counter E.M.F., or IE_2, and the intake is the product of the current and the applied E.M.F., or IE_1, the efficiency is,

$$\frac{IE_2}{IE_1} = \frac{E_2}{E_1} = \frac{450}{500} = 90\%.$$

EXAMPLE. — Determine at what efficiency a motor gives its greatest output per minute.

SOLUTION. — Since the current through a motor is $\frac{E_1 - E_2}{R}$, in which E_1 and E_2 are the applied and counter E.M.F's. respectively, and R is its resistance, the output is $E_2 I = \frac{E_2(E_1 - E_2)}{R}$. In this E_1 and R are constants. Obviously, then, the numerator, consisting of this product of a variable and the difference between a constant and the variable, will be the greatest when $E_2 = \frac{1}{2}E_1$. In other words, the output is at the greatest rate when the counter E.M.F. is $\frac{1}{2}$ the applied E.M.F., and the efficiency, therefore, 50%. To illustrate,

Suppose the applied E.M.F., $E_1 = 100$ volts.

Suppose the counter E.M.F., $E_2 = 50$ volts.

Then the output will be represented by

$$E_2 (E_1 - E_2) = 50 (100 - 50) = 2500.$$

Let the speed increase, so that the counter $E_2 = 60$ volts. The output is now

$$60 (100 - 60) = 2400.$$

Again, let a decrease of speed reduce the counter E.M.F., E_2, to 40 volts. Then the output is

$$40 (100 - 40) = 2400.$$

This shows that the greatest rate of work is done at an efficiency of 50 per cent, but is modified in practice.

24. Original Problems. — 1. A voltmeter connected across a Buckeye lamp read 109.5 volts when 0.6 ampere was flowing through the lamp. How many joules of energy are used by the lamp per second, and how many

ergs? How many such lamps can be maintained per H.P.? How many calories of heat are developed per second in each lamp? If the lamp has a candle-power of 14, what is the efficiency in watts per c.p.?

$W =$ 65.7 joules = 657,000,000 ergs = 15.7 cal.
Eff. = 4.7 watts per c.p.
11 lamps per H.P.

2. I wish to supply 50 lamps at 110 volts, resistance of each being 220 ohms hot. The line loss is 5%, the commercial efficiency of the generator can be counted as 92%, and the engine and belt losses are 15%; what I.H.P. will be necessary to operate the plant?

I.H.P. = 5 (strictly 4.9).

3. Find the H.P. required to furnish 40 arc lamps in series with 7 amperes of current, the resistance of each lamp being 8 ohms, line 25 ohms. How many watts are necessary per lamp? Assuming a normal candle-power of 1200 for each lamp, find the efficiency in watts per c.p. Compare last result with the efficiency in 1.

H.P. = 22.6.
Eff. = 0.326 watt per c.p.

4. An engine on full load indicates 12.5 H.P. Assuming engine and belt losses to be 12%, and the commercial efficiency of the dynamo 90%, line loss 5%, how many 110 volt incandescent lamps constitute the load? Also, what is the voltage at the brushes? How many incandescent arcs could be supplied at 4 amperes each?

No. lamps, $n = 127$.
Brush volts = 115.8.
Inc. arcs, $n = 16$.

5. The resistance of a certain dynamo machine's armature is 0.016 ohm, that of the external circuit 0.757 ohm. The power required to operate the machine is 7.604 H.P., and the current produced is 83.7 amperes. What are the efficiency of conversion and the commercial efficiency of the machine? $C = 95.5\%$.

Com. Eff. = 93.5%.

6. A circuit consists of 100 incandescent lamps arranged in 20 parallel groups, each group containing 5 lamps in series. The voltage between the main wires at the center of the lamp system is 550, and the resistance of each lamp is 212 ohms, average. Find the watts and ergs used per lamp. Also the amount of current flowing, and the efficiency of transmission if the line resistance is 8 ohms.

Watts per lamp = 57 = 57×10^7 ergs per sec.
Eff. of trans. = 87%.
$I = 10.37$ amperes.

7. If the following set of conditions are representative, how many 110 volt incandescent lamps can be supplied per I.H.P. of the engine? Lamps 100, drop in line 6.5 volts, resistance of fields, shunt wound, 25 ohms, of armature 0.025 ohm; C of generator 96%, engine and belt losses 15%? Find the plant efficiency.

Lamps per I.H.P. = 9.
Plant eff. = 73%.

8. Compare the external heat with the internal in the following cases; also determine the efficiency in each case; the battery resistance is equal to 7 feet of a certain size of wire, and for the first case has its terminals con-

nected through 2 feet of the wire; for the second case the battery terminals are connected through 53 feet of the wire.

$$(1)\ \frac{H(\text{ex.})}{H(\text{in.})} = \frac{2}{7};\quad \text{eff.} = \frac{2}{2+7} = 22.2\%.$$

$$(2)\ \frac{H(\text{ex.})}{H(\text{in.})} = \frac{53}{7};\quad \text{eff.} = \frac{53}{53+7} = 88.3\%.$$

9. A current of 0.75 ampere is sent for 5 minutes through a column of mercury whose resistance is 0.47 ohm, and which weighs 20.25 grams, and has a specific heat of 0.0332. Find the rise of temperature, assuming no heat lost by radiation. — *Day.* $t = 28°$ C.

10. Assuming the rate of radiation previously given, find how hot an iron wire $\frac{1}{8}$ inch in diameter and 100 feet long will become in the air when it carries 20 amperes. $t = 47°$.

11. What must be the diameter of a German silver wire 50 feet long to carry 5 amperes so that it may not rise to exceed 30° above the temperature of the surrounding air? $d = 74$ mils = No. 13.

12. How many amperes of current may a copper wire 64 mils diameter and 500 feet long carry so as not to exceed 25° above the temperature of the air?

$I = 13$ amperes.

13. A coil of platinum wire is immersed in a tank holding 3 quarts of water which is constantly stirred while 4 amperes of current pass through the wire which has a length of 20 feet, and diameter of 40 mils. The initial temperature of the water is 15°. Assuming no loss by radiation, what will be the temperature of the water after 10 minutes? $t = 15.56°$ C.

14. Find the gauge of a zinc wire to be inserted in a circuit to carry 250 amperes as a maximum current, when zinc fuses at 422° C. $d = 270$ mils = No. 2.

15. What size of copper wire must be used for the protecting fuse of a circuit designed to carry normally 300 amperes, allowing temporary over-loading of 40%? $d = 0.48$ cm. = 188 mils = No. 5.

16. What is the fusion temperature of an alloy when a piece of gauge number 16 fuses at 20 amperes, knowing that the specific resistance is 24.66? $t = 1800°$ C.

17. If a certain machine when run at 2200 r.p.m. generates 116 volts at the brushes, and supplies 110 volts and 22 amperes of current at a certain distance, how many times as far will it furnish the same power when run at 2300 r.p.m.? 1.9 times as far.

18. The poles of a dynamo whose resistance is 0.08 ohm are in the first place joined through 4 incandescent lamps in series, in the second place through 4 in parallel. The resistance of each lamp is 50 ohms, and the mechanical energy supplied to the machine is the same in both cases. Compare the amounts of heat developed in the machine? — *Day.* The lamp resistance is here wrongly assumed constant.

$$\frac{H_2}{H_1} = \frac{16}{1}.$$

19. A circuit contains 20 incandescent 220 ohm lamps in parallel, the lead wires having a resistance of 1 ohm and the machine 0.1 ohm. Now a wire whose resistance is 0.5 ohm is connected across the leads at the lamps,

while the E.M.F. at the brushes is 120. Compare the current in the lamps with that in the wire. How does the heat in the machine compare in each case with that in the external circuit? How much greater load is placed on the engine by the wire?

Normal $I = 10$ amperes.
With wire, $I = 81$ amperes, 3.5 in lamps.
Normal heat in machine $H_1 = 10$ watts.
2d heat in machine $H_2 = 656$ watts.
Load with wire 8 times normal.

20. A dynamo and a motor are exactly alike, each having a normal E.M.F. of 110 volts when running at 2000 r.p.m. The dynamo is kept running at the normal speed, but the load of the motor is such that it runs at 1500 revolutions per minute. What must be its efficiency at this load and speed?

Counter E.M.F = 82.5 volts.
Eff. = 75%.

21. What H.P. must a hoisting motor be capable of developing when 900 pounds are to be lifted 80 feet per minute, the cage being counterbalanced by a weight suspended from the drum, and making an allowance of 50% of its power for losses due to friction in all parts?

H.P. = 4.37.

22. What H.P. would be required if the cage were drawn at the same speed up an incline in which there is a rise of 10 feet for every 10 feet in a horizontal direction, in other words when the grade is 100%?

SOLUTION. — Here the ratio of the vertical lift to the slope must be taken as the ratio of power required to that for a vertical lift. Slope $= \sqrt{10^2 + 10^2} = 14.14$. Therefore the power will be $4.37 \times \frac{10}{14.14} = 3$ H.P.

23. A motor is to operate a pump lifting 20,000 gallons per hour 500 feet high. Allowing an efficiency of 60% in the pumping system, and given that 1 gallon weighs 8.33 lbs., how many H.P. must the motor develop? Also putting its efficiency at 90% and operating it on a 220 volt circuit, how many amperes of current will it require?

70 = H.P. developed.
78 = H.P. supplied.
$I = 265$ amperes.

24. An electric car with its load weighs 12 tons and is to maintain a speed of 10 miles an hour on a grade of 2%; that is, 2 feet rise for every 100 measured horizontally. Allowing a traction effort of 25 lbs. per ton, how much power is necessary at the axles, and assuming an efficiency of 75% in motor and gearing, how many amperes will it take from a 500 volt circuit?

Power at axles = 21 H.P.
$I = 41.8$ amperes.

V.

WIRING FOR LIGHT AND POWER.

25. **Drop of Potential and Size of Leads.** — The simplest case of wiring is that for *arc lamps* which are all in series, 70 to 80 volts being required for each lamp. The machine at the station automatically changes the pressure to suit the number of lamps, while the current is kept constant at 5 to 7 amperes, depending on the system used. This is therefore a *constant current* system.

The resistance of the wire is generally taken so that not to exceed 10% of the dynamo volts will be required to pass the current through it.

Example. — A certain circuit carries 40 T.H. arc lamps in series, inclosed type; each lamp requires 75 volts, the current being 5.5 amperes. Allowing a line drop of 5% what must be the difference of potential at the brushes, and what size of wire must be used, its length being 3 miles?

$E = 3157.8$ volts.

$R = 28.69$ ohms.

Wire is No. 12, A.W.G. or B. & S.

Incandescent arc lamps are those designed to work in parallel on the same circuit as incandescent lamps. They are used for both interior and exterior lighting, and wound for 110 or 50 volts. The calculations are the same in

this case as for incandescent work, and so will not be treated separately.

Although for incandescent lighting and power on the *parallel*, or *constant potential* system, the wiring often becomes complex and the exact distribution of copper in the various parts of the net-work a matter for much study, yet the principles which govern the calculations are the same as for simple circuits, and are all based on Ohm's law. The problem may be to calculate the leads and lamp circuits leading from street mains or transformers to the lamps, or it may include the mains and all the wiring of a complex system.

Example. — The street mains are kept at the constant potential of 116 volts. Lamp leads are run out to the center of a system of 150 incandescent lamps. The voltage at the point from which the lamp circuits leave the leads is 112. Calculate the resistance per foot of the wire to be used if the leads are 100 feet long, and obtain from the table the corresponding gauge number.

Solution. — Line drop $= 116 - 112 = 4$ volts.

I for 150 lamps $= 150 \times \frac{1}{2} = 75$ amperes.

Hence R, from Ohm's law, $= \frac{E}{I} = \frac{4}{75} = 0.053$ ohm.

Leads contain 200 feet of wire; hence the resistance per foot of the wire to be used is

$R = 0.053 \div 200 = 0.000266$ ohm; and per 1000 feet, $R = 0.000266 \times 1000 = 0.266$. From the table this corresponds to No. 4 A.W.G.

EXAMPLE. — Determine the size of the wire for the lamp circuits in the above problem assuming all to be of equal length, averaging 30 feet from the junction with leads, and carrying 10 lamps each.

SOLUTION.—Drop in lamp circuits $= 112 - 110 = 2$ volts.

$$\text{Current in each} = 10 \times \tfrac{1}{2} = 5 \text{ amperes.}$$

$$R \text{ of each} = \frac{E}{I} = \frac{2}{5} = 0.4 \text{ ohm.}$$

$$\text{Length of wire in each circuit} = 2 \times 30 = 60 \text{ ft.}$$

Hence per 1000 feet,

$$R = 0.4 \times \tfrac{1000}{60} = 6.66 \text{ ohms} = \text{No. 18 A.W.G.}$$

The rules for correct wiring do not permit wire smaller than No. 14 to be used. If 14 is used here the resistance for 60 feet $= 0.158$ ohm, and the drop will become $E = 0.158 \times 5 = 0.79$ volt. This means that the lamps will burn at $112 - 0.79 = 111.2$ volts, or else the voltage at the street mains must be reduced about 1 volt so that the lamps will burn at 110 volts.

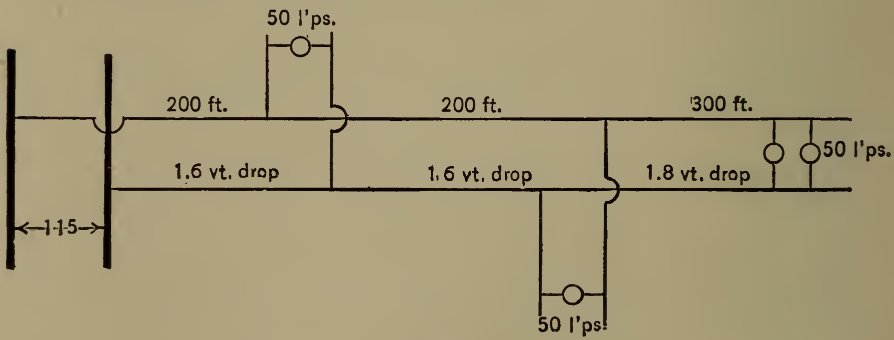


Fig. 4.

EXAMPLE. — Let the diagram, Fig. 4. represent the conditions of a circuit, to solve for the size of the various portions of the leads.

Solution. — The current required by the first group of 50 lamps, each of which has a resistance of 220 ohms, will be

$$I_1 = \frac{115 - 1.6}{220 \div 50} = 25.7 \text{ amperes.}$$

The second, $I_2 = \dfrac{113.4 - 1.6}{220 \div 50} = 25.41$ amperes.

The third, $I_3 = \dfrac{111.8 - 1.8}{220 \div 50} = 25$ amperes.

Therefore the total current flowing through the first section of the leads is

$$I' = 25.7 + 25.41 + 25 = 76.1 \text{ amperes.}$$

Total current flowing through the second,

$$I'' = 25.41 + 25 = 50.41 \text{ amperes.}$$

In the last section,

$$I''' = 25 \text{ amperes,}$$

Hence the resistance of the first section is

$$R_1 = \frac{E}{I} = \frac{1.6}{76.1} = 0.021 \text{ ohm.}$$

This is for $200 \times 2 = 400$ feet of wire.

The resistance for 1000 feet then is

$$R = \frac{0.021 \times 1000}{400} = 0.052 \text{ ohm.}$$

Table No. 2 gives this to be about No. 000 wire.

The resistance of the second section is

$$R_2 = \frac{1.6}{50.41} = 0.0317 \text{ ohm for 400 feet.}$$

2. Table of Wiring Constants.

B. & S. Gauge No.	Diameter Mils.	Area Arc Mils.	Wt., Lbs. per 1000 ft.			R, Ohms per 1000 ft.	Safe Carrying Capacity, — Amperes.		
			Bare.	Insulated. Weather-Proof	Insulated. Single Cot. Cov.		Bare Outside Wire.	House Wire.	Magnetic Wire.
0000	460	211,600	640.5	730		.05075	620	175	
000	410	167,805	508.0	588		.06400	525	147	
00	365	133,079	402.8	476		.08070	538	124	
0	325	105,593	319.6	385		.10171	369	104	
1	289	83,695	253.4	308		.12832	309	87	
2	258	66,373	201.0	250		.16181	260	73	
3	229	52,634	159.3	200		.20404	219	62	
4	204	41,743	126.4	160		.25729	183	49	
5	182	33,102	100.2	133		.32444	154	44	
6	162	26,251	79.46	111		.40914	130	37	
7	144	20,817	63.01			.51593	109	31	
8	128	16,510	49.98	72		.65053	92	26	
9	114	13,094	39.64			.82022	77	22	
10	102	10,382	31.43	51		1.03458	65	18	
11	91	8,234	24.93			1.30434	54	15.4	
12	81	6,530	19.77	33	23.8	1.64474	46	13	6.5
13	72	5,178	15.68		18.2	2.07400	38	11	5.2
14	64	4,107	12.43	25	14.7	2.68044	32	9	4.1
15	57	3,257	9.86		11.5	3.29782	27	7.6	3.3
16	51	2,583	7.82	15	9.1	4.15812	23	6.4	2.6
17	45	2,048	6.20		7.2	5.24363	19	5.4	2.0
18	40	1,624	4.92	11	5.7	6.61208	16	4.5	1.6
19	35	1,252	3.79		4.5	8.562			1.3
20	32	1,022	3.09		3.6	10.498			1.0
21	28	810	2.45		2.8	13.237			.8
22	25	642	1.94		2.2	16.690			.6
23	23	509	1.54		1.8	21.050			.5
24	20	404	1.22		1.4	26.547			.4

Per 1000 ft.,

$$R = \frac{0.0317 \times 1000}{400} = 0.079 \text{ ohm, or No. 00 wire.}$$

The resistance of the third section is

$$R_3 = \frac{1.8}{25} = 0.072 \text{ ohm for 600 feet.}$$

Per 1000 ft.,

$$R = \frac{0.072 \times 1000}{600} = 0.120 \text{ ohm, or No. 1 wire.}$$

The practice in wiring is so to distribute the drop of potential, by adjusting the resistance of the circuits, that it decreases from the dynamo outwards to the lamps. For example, let a small plant supply a circuit so as to lose 6 volts. We may wire for 3 volts drop in the mains, 2 volts in the leads, and 1 volt in the lamp circuits.

26. Wiring Formulæ and Diagrams. — For greater convenience in calculation we may apply a certain formula for finding the number of circular mils required in a wire to supply a given number of lamps, at any distance required, for a given drop of potential.

Example. — Let it be required to derive the formula for the circular mils required in any portion of a circuit when the distance and the number of lamps and per cent of drop are given.

Solution. — We have already for resistance (15),

$$R = K\frac{l}{d^2} = 10.79\frac{l}{d^2}.$$

However, 10.79 is here used as more nearly representing the specific resistance of ordinary commercial copper

than 9.8, the specific resistance of pure copper which was used previously in this book. By transposition of the above formula we have for circular mils,

$$d^2 = \frac{10.79\,l}{R}. \tag{29}$$

We also have for the resistance of any number of lamps n in parallel when the resistance of each one is r,

$$r_1 = \frac{r\,(\text{hot})}{n}. \tag{30}$$

Since resistances are proportional to the drops of E.M.F. in them,

$$\frac{R\,(\text{line})}{r_1\,(\text{lamps})} = \frac{\%\ \text{drop in line}}{100 - \%}. \tag{31}$$

Here $100 - \%$ drop in line = per cent drop in lamps. The per cent is calculated on the highest voltage of the system as a base. By transposition of (31),

$$R = \frac{r_1 \times \%}{100 - \%}. \tag{32}$$

Putting in (32) the value of r_1 from (30),

$$R = \frac{r\,(\text{hot})}{n} \times \frac{\%}{100 - \%}. \tag{33}$$

Equation (33) gives the line resistance in terms of the resistance of one lamp hot, the number of lamps, and the per cent drop. Having thus the resistance and knowing the length of the wire, the size could be determined as we have already done. But to make the work still more complete, place this value of R in (29); then for circular mils we have

$$d^2 = \frac{10.79\,l}{\dfrac{r\,(\text{hot})}{n} \times \dfrac{\%}{100 - \%}}.$$

Simplifying, $$d^2 = \frac{10.79\,l \times n}{r\,(\text{hot})} \times \frac{100 - \%}{\%}. \qquad (34)$$

But l the length is twice the measured distance D; that is, $l = 2D$. Hence, placing this in (34),

$$^{*}d^2 = \frac{21.58 \times (D \times n)}{r\,(\text{hot})} \times \frac{100 - \%}{\%}. \qquad (35)$$

The product Dn is called the "lamp feet" of the circuit, being the product of the distance in feet by the number of lamps. r is 220 for 110 volt lamps and 50 for 50 volts.

EXAMPLE. — A circuit is to be run 400 feet for 150 lamps at a loss of 5 per cent. The lamps are rated at 110 volts and $\frac{1}{2}$ ampere. Determine the size of the main wire. — *Badt.*

SOLUTION. — Applying directly (35),

$$d^2 = \frac{21.58 \times 400 \times 150}{220} \times \frac{100 - 5}{5} = 111{,}823 \text{ cir. mils.}$$

This corresponds to No. 0 wire A.W.G. or B. & S.

It is evident that as long as 110 volt lamps are used $\frac{21.58}{220}$, or if 50 volt lamps are used $\frac{21.58}{50}$ will be constant, and the value of the fraction may be so used in the wiring formula. Then (35) will become for 110 volt lamps

$$d^2 = 0.098 \times Dn \times \frac{100 - \%}{\%}. \qquad (36)$$

* Instead of 21.58 in this equation for carbon lamps, use the constant 12 for 30-watt tungstens, constant 16 for 40-watt, and constant 24 for 60-watt lamps.

And if 50 volt lamps are used

$$d^2 = 0.412 \times Dn \times \frac{100 - \%}{\%}. \tag{37}$$

Furthermore, if $\frac{100 - \%}{\%}$ be worked out for the per cents drop ordinarily met in practice, and its value combined with the constant just found, the formula may thus be very much simplified. Thus suppose a drop of 5 per cent. Then $\frac{100 - \%}{\%} = 19$. This multiplied by 0.098 gives 1.86 as the constant for 110 volt 220 ohm lamps at 5 per cent drop. The only variables are now D and n. For 50 volt lamps the total constant will be $0.412 \times 19 = 7.828$ at 5 per cent; and (37) will be, at 5 per cent,

$$d^2 = 7.828 \times Dn.$$

For 110 volt lamps there are set down the complete simplified equations for various per cents up to 10 per cent.

$$\left.\begin{array}{l}
\text{For } 1\ \%\ \text{drop, } d^2 = 9.7 \times Dn \\
\text{For } 1\tfrac{1}{2}\%\ \text{drop, } d^2 = 6.44 \times Dn \\
\text{For } 2\ \%\ \text{drop, } d^2 = 4.8 \times Dn \\
\text{For } 2\tfrac{1}{2}\%\ \text{drop, } d^2 = 3.82 \times Dn \\
\text{For } 3\ \%\ \text{drop, } d^2 = 3.17 \times Dn \\
\text{For } 3\tfrac{1}{2}\%\ \text{drop, } d^2 = 2.70 \times Dn \\
\text{For } 4\ \%\ \text{drop, } d^2 = 2.35 \times Dn \\
\text{For } 5\ \%\ \text{drop, } d^2 = 1.86 \times Dn \\
\text{For } 6\ \%\ \text{drop, } d^2 = 1.54 \times Dn \\
\text{For } 7\ \%\ \text{drop, } d^2 = 1.30 \times Dn \\
\text{For } 8\ \%\ \text{drop, } d^2 = 1.13 \times Dn \\
\text{For } 9\ \%\ \text{drop, } d^2 = 0.99 \times Dn \\
\text{For } 10\ \%\ \text{drop, } d^2 = 0.88 \times Dn
\end{array}\right\} \quad (38)^*$$

* For 30-watt tungsten lamps multiply the constants in these equations by 0.55; for 40-watt, multiply by 0.75; for 60-watt, multiply by 1.1.

Example. — Calculate the size of the leads for 200 lamps at 300 feet for 4 per cent drop. Lamps always 110 volts unless otherwise specified.

Solution. — $d^2 = 2.35 \times Dn = 2.35 \times 300 \times 200$
$= 141{,}000$ cir. mils = No. 00, A.W.G.

The drop should be made as small as it can be consistent with economy in the cost of conductors; (1) because large drops are detrimental to good regulation; (2) because large drops mean the same percentage waste of the energy of the generator. The former is the more important from the consumer's standpoint, while the latter reason appeals the more directly and strongly to the producer.

Example. — Suppose a circuit has 100 lamps and the drop on mains and leads is 15 volts. What will be the voltage of the rest of the lamps when 50 are turned out? Determine the same for 5 volts drop.

Solution. — Assume the lamps to have 110 volts pressure, average. Drop $E = IR$; when 50 are cut out I is only half as large as before; hence, since R is constant, E the drop will be only half as much as before, or $7\frac{1}{2}$ volts. Therefore, until the machines are regulated to suit the new conditions the remaining fifty lamps will run at $110 + 7\frac{1}{2} = 117\frac{1}{2}$ volts, which is excessive. Likewise any small change in the number of lamps will be accompanied by a proportional variation of voltage in the rest.

If the total drop at full load be only 5%, then the other 50 lamps will run at $110 + \frac{1}{2}$ of $5 = 112\frac{1}{2}$ volts when one-half the load is taken off.

It is apparent then that uniformity of light under varying load can best be secured by having the full-load line drop as small as can be made consistent with economy in the cost of wire.

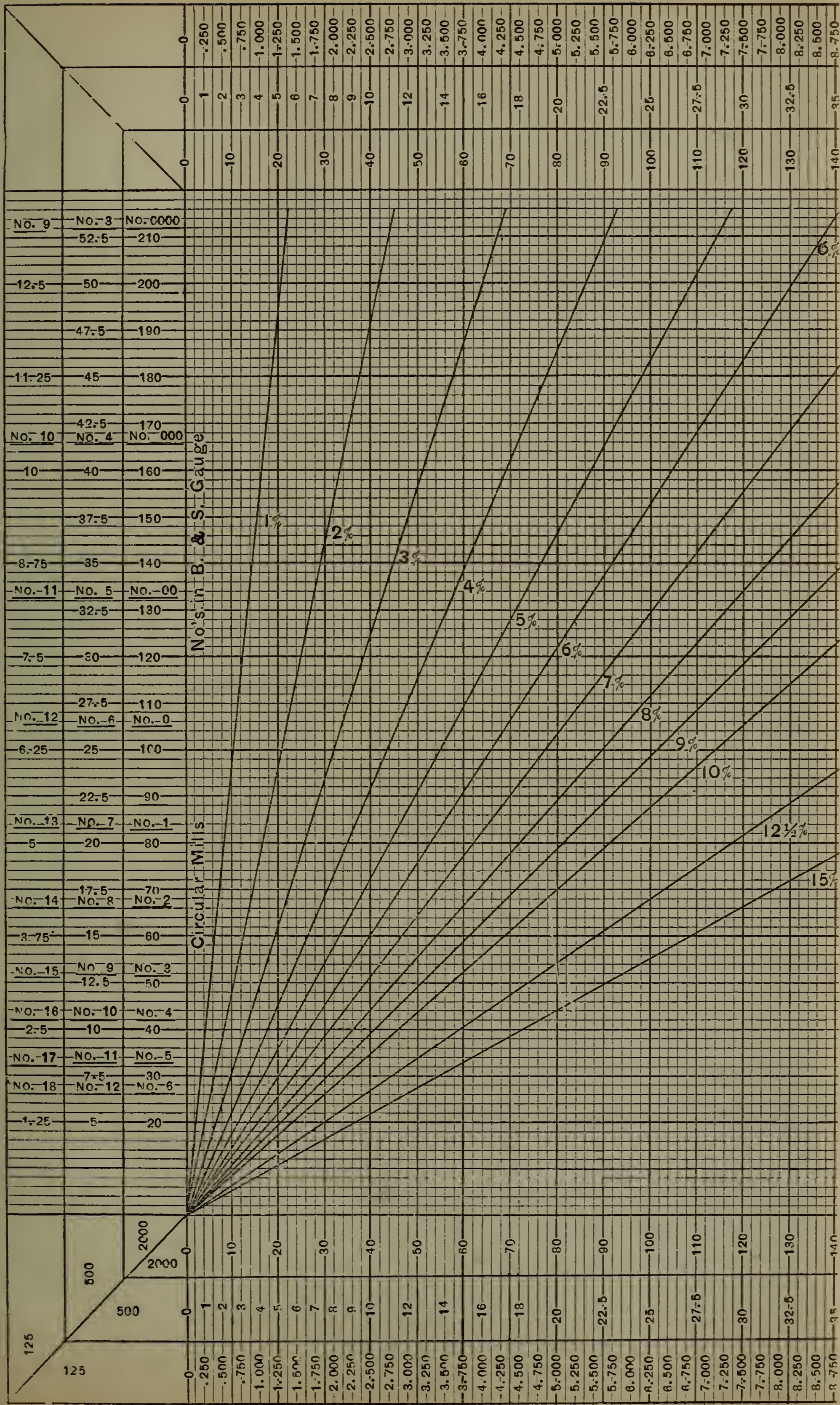
From this it should also be clear that the drop should be divided so as to decrease from the generator out through *mains*, *leads*, and *lamp circuits*.

Wiring Diagrams. — It will often be convenient to consult the wiring diagrams for installing the line for 110 and 55 volt lamps, Tables 3 and 4, instead of applying the formula in calculation. In these there are three horizontal and three vertical columns. The horizontal ones mean "lamp feet." The vertical ones mean "circular mils," except the underscored numbers, which mean the gauge number, B. & S., or A. W. G. The numbers in both sets of columns are so many *thousands*. In consulting the diagrams always use corresponding columns, inside-inside, middle-middle, outside-outside.

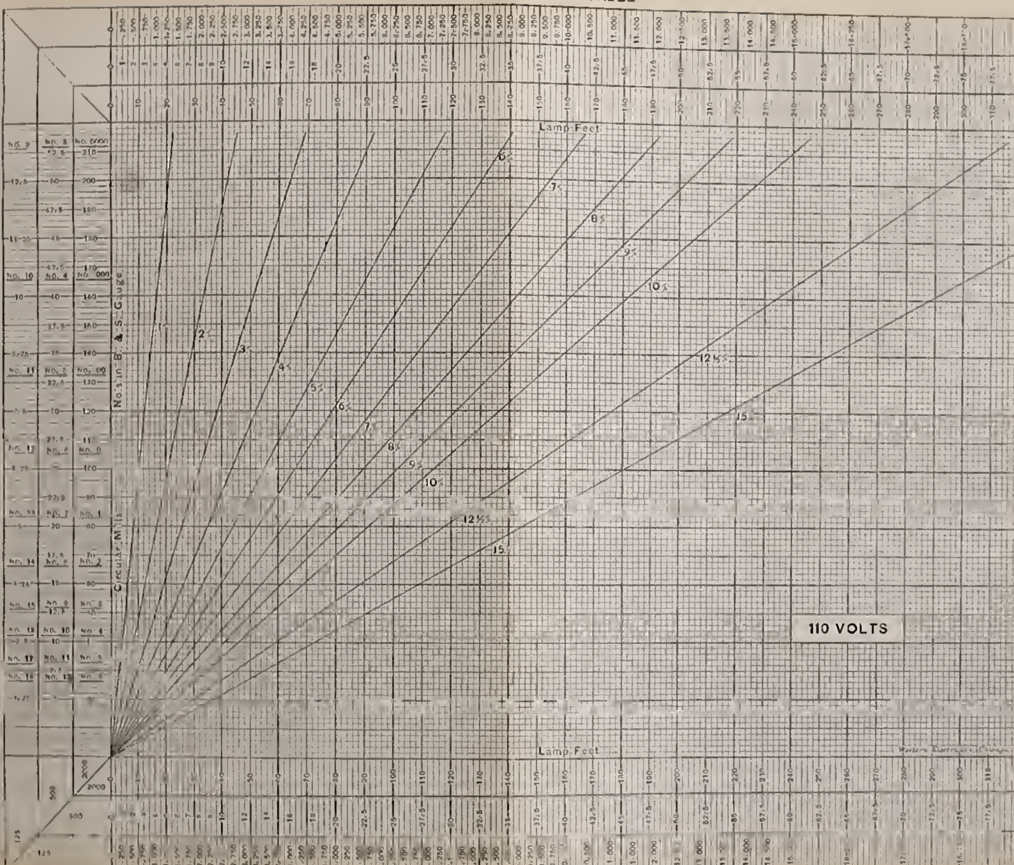
Example. — Use the diagram, and determine the size of wire for 150 lamps, 110 volts, at a distance of 200 feet, drop 5%.

Solution. — $Dn = 200 \times 150 = 30{,}000$ lamp feet. Find 30 in the inside horizontal column, follow up vertical line to intersection of 5 per cent radial line, then move left to inside vertical column, and obtain 54,000 cir. mils, which is between No. 3 and No. 2, nearer No. 3.

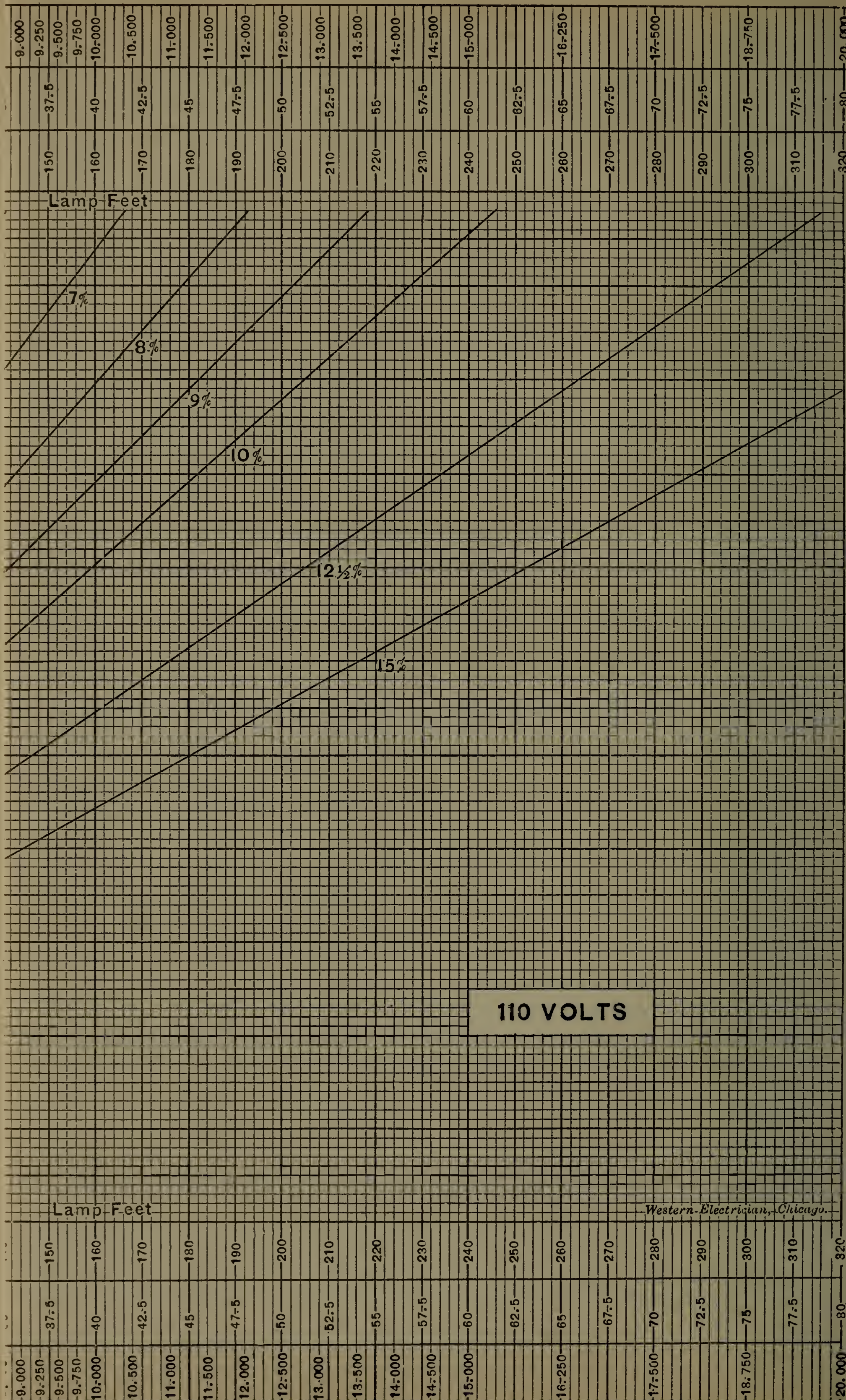
If the same number of lamps is to be wired for 55 volts at the same drop, the above process on the 55 volt diagram obtains 222,000 cir. mils, or about 2 No. 0 wires


No's in B. & S. Gauge
Circular Mills
1%
2%
3%
4%
5%
6%
7%
8%
9%
10%
12½%
15%
No. 9
No. 3
No. 0000
No. 10
No. 4
No. 000
No. 11
No. 5
No. 00
No. 12
No. 6
No. 0
No. 13
No. 7
No. 1
No. 14
No. 8
No. 2
No. 15
No. 9
No. 3
No. 16
No. 10
No. 4
No. 17
No. 11
No. 5
No. 18
No. 12
No. 6
2000
500
125

INCANDESCENT WIRING TABLE



T WIRING TABLE



in parallel, thus showing the poor economy of wiring 55 volt lamps at a low per cent drop.

If the drop be made 10 per cent for 55 volt lamps the wire will be No. 0.

In case the lamp feet are too large to be found in one of the horizontal columns, divide by 10, find the size of wire for this number of lamp feet, then multiply the circular mils so found by 10.

*EXAMPLE. — Find the circular mils for a main to supply 500 lamps at 2000 feet, at a loss of 10%.

SOLUTION.—$Dn = 2000 \times 500 = 1{,}000{,}000$. One-tenth of this is 100,000. Find the circular mils corresponding to be 88,000. Multiply this by 10, making $d^2 = 880{,}000$ cir. mils. This is equivalent to 8 No. 0 wires in parallel.

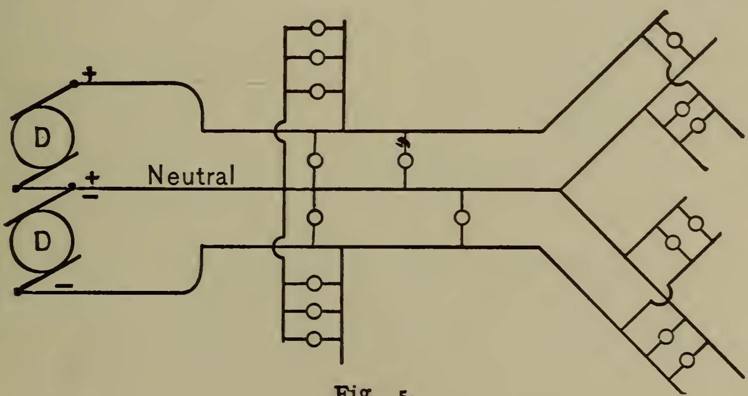
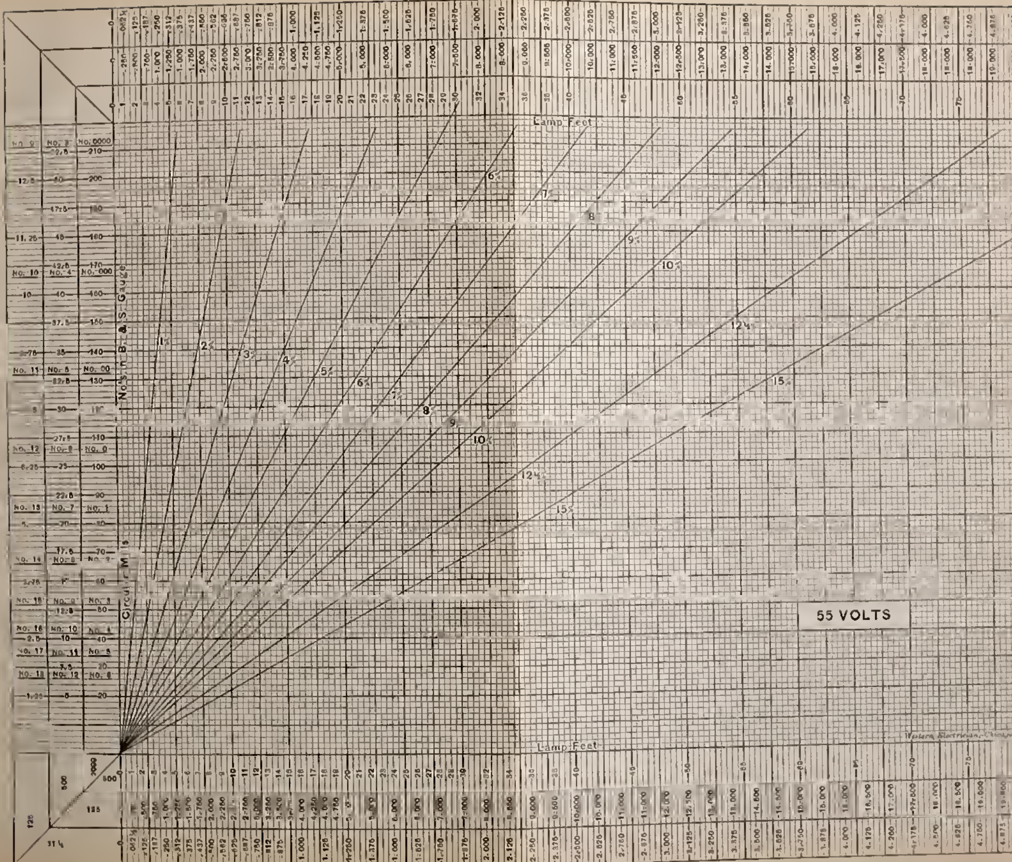


Fig. 5.

27. **The Three-Wire System.** — By the three-wire system is meant one in which two dynamos are connected in series, one terminal of each being connected respectively to one of the wires of the circuit, while the common junction of the machines also connects with a third or middle

* In applying the 110-volt diagram to tungsten lamps, find the circular mils as for carbon lamps, then use the multipliers suggested on page 74 for tungstens.

INCANDESCENT WIRING TABLE



wire of the circuit. The system therefore requires three wires for its operation. The voltage between the outside wires is twice the voltage of a single machine, while that between the outside wires and the middle, or neutral wire, is equal to that of each machine. When the system is in operation two lamps are in series between the outside wires, thus requiring one-half the current which the same number of lamps would require on the two-wire system. The diagram, Fig. 5, will make the details of the circuits clear.

The three wires are usually continued out to the lamp circuits, and these also often have three wires where there is a very large number of lamps on each circuit. The economy in wire is apparent from the following considerations.

The voltage between the outside wires is double that in the two-wire system and, as has been shown, the current is one-half. Hence for a given per cent drop the volts drop will be double that for two wires. The resistance of the wire is the volts drop divided by the current, or $R = \frac{E_3}{I_3}$, where the subscripts denote the kind of system. But $E_3 = 2E_2$, and $I_3 = \frac{1}{2}I_2$. Hence $R_3 = \frac{2E_2}{\frac{1}{2}I_2} = 4\frac{E_2}{I_2}$. That is to say, the resistance for the three-wire system is 4 times the resistance for the two-wire system. Therefore the cross section of each wire is $\frac{1}{4}$ that of each wire in the two-wire system.

To determine the wire for this system, then, apply the rules for the two-wire circuit, and take $\frac{1}{4}$ the circular mils so found.

EXAMPLE. — Find the size of wire for 200 lamps, 110 volts, at 200 feet, for 5 per cent drop.

SOLUTION. — From (38), $d^2 = 1.86 \times Dn$.

Therefore

$$d^2 = 1.86 \times 200 \times 200 = 74{,}400 \text{ cir. mils.}$$

$\frac{1}{4}$ of 74,400 = 18,600 cir. mils for 3 wires, No. 8, nearest.

The middle wire could usually be made smaller; but should the number of lamps on the two sides of the circuit be unequal, it has to carry the excess back to the machine. It is therefore nearly always made the same size as the outside wires. The total copper then in the three-wire system is $\frac{3}{8}$ as much as for the same number of lamps on the two-wire system; for each lead in the former is $\frac{1}{4}$ of each in the latter, and 3 leads make $\frac{3}{4}$ of each, or $\frac{3}{8}$ of both leads in the latter. That is, if a certain system could be installed using 2 wires for $500.00, it could be put in with 3 wires for $\frac{3}{8}$ of $500.00 = $187.50. This is partly offset, however, by the extra cost of installing, and by the necessity of using two machines.

Several different methods * have been devised to overcome the disadvantage of using two machines, though they are but little used, if at all. An armature may have two independent windings and two commutators to which the circuits may then be connected, as with two machines. In the second method, the generator is wound to give 220 or 440 volts, and a storage battery is connected across the wires; the neutral wire is then joined to the center of the battery. Again, a third brush may be set for the neutral wire to give half the potential of the machine.

* See Crocker's *Electric Lighting*, Vol. II., Chap. IV., for complete description and diagrams of the various three-wire systems.

28. Original Problems. — 1. By the fall of potential method find the gauge number of the wire to be used as leads from house switchboard to lamp circuit junction box 65 feet, to supply 200 lamps, 110 volts, at $2\frac{1}{2}$ volts drop.

Wire No. 3 B. & S.

2. Suppose the lamps in the above problem were 50 volt lamps wired for the same drop; what size of leads must be installed? Wire No. 0 B. & S.

3. Determine the wire for the following system. From the house switchboard a set of leads is run north 100 feet where a branch is taken off to a distributing point for 20 lights; 75 feet farther north another branch is run off from leads for 15 lights; 50 feet still farther north on leads is a junction box for 10 lights. The drop is $2\frac{1}{2}$ volts to first branch, $1\frac{1}{2}$ between the first and second, and 1 volt between second and third. Lamps are 110 volts.

Wire, first section, No. 7.
Wire, second section, No. 9.
Wire, third section, No. 13.

4. In the same way determine the size of the lamp circuit wires for a drop of 1 volt, if the lamps from the first junction are at an average distance of 50 feet, from the second 30 feet, and from the third 20 feet.

First	No. 10 B. & S.	
Second	No. 14 B. & S.	Nothing less than
Third	No. 14 B. & S.	14 to be used.

5. Use the formula for wiring, and calculate the mains

for 500 incandescent lamps, 110 volts, at a distance of 500 feet for a drop of 8% of the generator volts.

Wire 2 No. 00 in parallel.

6. The circuits for problem 5 run out from the distributing switchboard as follows: one circuit north 100 feet for 100 lamps; one south 50 feet for 200 lamps; one west 100 feet for 80 lamps; one east 150 feet for 80 lamps. Drop on leads $2\frac{1}{2}\%$. Determine the gauge number.

North No. 4.
South No. 4.
West No. 5.
East No. 3.

NOTE. — No. 4 wire could be used without any appreciable disturbance of the regulation, and in practice probably would be used in each branch.

7. Determine the lamp circuits in the last problem, drop $1\frac{1}{2}\%$, and draw complete diagram of the system, the following data being given. The *north branch* supplies from the junction point 10 circuits averaging 10 lamps each, distances from junction averaging 40 feet. The *south branch* connects with four junction boxes near together; from No. 1, 60 lamps are fed on 5 circuits averaging 50 feet long; No. 2, 30 lamps on 4 circuits averaging 25 feet; No. 3, 80 lamps on 8 circuits, 30 feet; No. 4, 30 lamps on 3 circuits, 40 feet. The *west branch* feeds two junctions; No. 1, 50 lamps on 4 circuits, 60 feet; No. 2, 30 lamps on 5 circuits, 50 feet. The *east branch* supplies two junction boxes; No. 1, 70 lamps on 7 circuits, 40 feet; No. 2, 50 lamps on 4 circuits, 50 feet.

North	No. 16, use No. 14.
South	No. 1, wire No. 14. No. 2, wire No. 18, use 14. No. 3, wire No. 17, use 14. No. 4, wire No. 14.
West	No. 1, wire No. 14. No. 2, wire No. 17, use 14.
East	No. 1, wire No. 16, use 14. No. 2, wire No. 14.

8. Use the wiring diagrams and verify the results obtained in problems 1 and 2, for 30-watt tungstens.

9. By means of the diagrams verify problems 3 and 4.

10. Verify the results in problems 5 and 6 by means of the diagrams. Also calculate for 40-watt tungstens.

11. Work out problem 7 by use of the wiring diagrams.

12. A town is to be wired, 3-wire system. Determine the sizes of wire to be used; also the machine volts and line drops. Street mains run from the station $\frac{1}{2}$ mile north on Main street to the intersection of Union street, and supplies east and west branches; the one east runs on Union street 500 ft., thence north on College street 250 ft.; the one west runs on Union street 500 ft., thence north on Congress street 250 ft. The Main street line also continues north 500 ft. to Washington street, whence it sends one branch east 500 ft., thence north on College street 250 ft.; also one branch west 500 ft., thence north on Congress street 250 ft. The Main street line also continues north from Washington street 400 ft. to State street, whence one branch runs east on State street 500

ft., and one west on State street 500 ft. There will be about 500 lamps, 110 volts, fed from the first junction on Main street, 300 on the east branch and 200 on the west branch. There will be probably 200 on the east branch at Washington street and 200 on the west branch. At State street there will be about 100 lamps fed east and 100 west. It is decided to lose 8% on the Main street feeder mains and 4% in the cross street lines, leaving 3% thence to lamps. See answers for %'s used.

First section main feeder requires 5 No. 0000 wires in parallel, 6% drop.
Second section 3 No. 000 wires in parallel, $1\frac{1}{4}$%.
Third section 2 No. 00 wires in parallel, $\frac{3}{4}$%.
First *east branch* No. 00 wire. First *west branch* No. 1 wire.
Second *east branch* and *west branch* No. 1 wire.
Third *east branch* and *west branch* No. 4 wire.
Machine volts $130 \times 2 = 260$.
Volts at branches 119.6.
Volts at junc. lamp leads 114.4.
Volts at beginning lamp circuits 111.8.
Drop on mains 10.4 volts.
Drop on branches 5 volts.

13. Continue problem 12 as follows, drawing complete diagram. Each one of the branches described above feeds into the center of a north and south lead wire running each way 125 feet to the center of lamp circuits. The first lead on College street supplies 150 lamps each side of feeding point, drop 2%. The second on College street supplies 75 lamps on the north and 125 on

the south of feeding point. The third supplies 75 lamps south and 25 north of feeding junction. The first on Congress street supplies 150 lamps north and 50 south of feeding point. The second on the same street feeds 100 lamps each way. The third, 40 lamps north and 60 south. Drop in all leads is 2%.

First, College street, No. 6 wire each way.
Second, College street, No. 7 S. and No. 10 N.
Third, College street, No. 10 S. and No. 14 N.
First, Congress street, No. 6 N. and No. 12 S.
Second, Congress street, No. 8 N. and S.
Third, Congress street, No. 12 N. and No. 10 S.

14. There are installed at a distance of 300 feet from the street mains 5 motors in a factory: 2 of 20 H.P. each, 1 of 40 H.P. and 2 of 50 H.P. each. What size of wire must be installed for 5% drop, the motors being 220 volt machines, and assuming the load not to exceed $\frac{2}{3}$ of the rated power?

2 No. 00 wires in parallel.

15. A motor of 200 H.P. is employed at full load to operate a line of counter shafting in a foundry. A special wire is erected, running from the general power station 500 feet distant. The E.M.F. at the station switchboard is 250 volts, and the motor requires 220 volts. Determine the size of wire to be installed.

2 No. 00 wires in parallel.

VI.

BATTERIES.

29. **Connection of Cells for Combined Output.** — Applying Ohm's law to the electric cell, the current delivered is

$$I = \frac{E}{R + r},$$

in which E is the electromotive force in volts measured between the electrodes when there is no circuit connected. R is the external, and r the internal resistance. E and r are called the "cell constants."

Example. — Through what external resistance must a cell be connected whose constants are $E = 1.5$ volts and $r = \frac{1}{5}$ ohm, so that 1 ampere may be obtained?

Solution. —

$$I = \frac{E}{R + r} = 1 = \frac{1.5}{R + 0.2}.$$

Hence $R + 0.2 = 1.5.$

Whence $R = 1.5 - 0.2 = 1.3$ ohms.

Example. — How much current will flow in a circuit of 5 ohms external resistance in which is placed a cell whose E.M.F. is 2 volts and internal resistance $\frac{1}{2}$ ohm?

$$I = \frac{E}{R + r} = \frac{2}{5 + 0.5} = 0.363 \text{ ampere.}$$

Series Connection. — Whenever any number of E.M.F.'s are connected together in series, the total E.M.F. is their

sum if all are positive in the same direction; or if some are reversed with respect to the rest, it is the difference between the sum of the positive and the sum of the negative E.M.F.'s. The total resistance is the sum of all the resistances in series. Therefore if any number of similar cells n be joined in *series*, the current flowing through any external resistance R will be

$$I = \frac{nE}{R + nr}. \tag{39}$$

EXAMPLE. — Draw a diagram of connections for 4 cells in series through an external resistance of 30 ohms, and calculate the current flowing when the cell constants are 1.8 volts and 1 ohm.

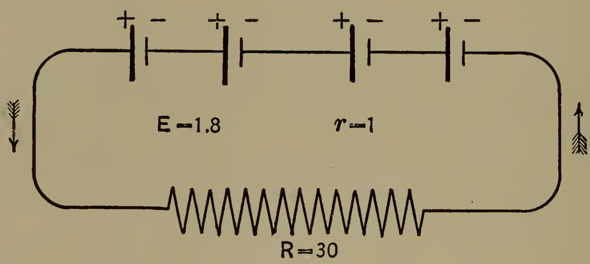


Fig. 6.

$$I = \frac{nE}{R + nr} = \frac{4 \times 1.8}{30 + 4 \times 1} = 0.212 \text{ ampere.}$$

For only one cell,

$$I = \frac{1.8}{30 + 1} = 0.058 \text{ ampere,}$$

or a little more than $\frac{1}{4}$ as much as 4 cells in series.

Multiple Connection. — When any number of equal E.M.F.'s are placed in multiple arc, or parallel, the total E.M.F. is that of a single one, the effect of such an

arrangement being merely to reduce the internal resistance according to the law expressed in Chapter III. Therefore when any number of cells m are connected in *parallel*, the current flowing in an external resistance R will be

$$I = \frac{E}{R + \frac{r}{m}}. \qquad (40)$$

EXAMPLE. — Draw a diagram of connections for 4 cells in parallel through an external resistance of 30 ohms, and calculate the current furnished by the battery thus formed when the cell constants are 1.8 volts and 1 ohm.

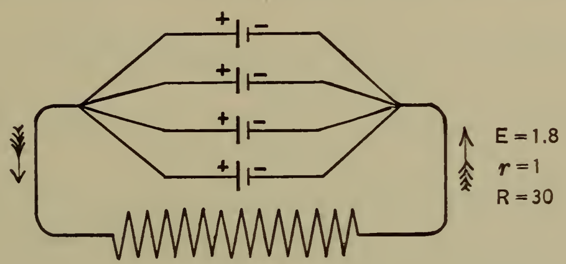


Fig. 7.

$$I = \frac{E}{R + \frac{r}{m}} = \frac{1.8}{30 + \frac{1}{4}} = 0.059 \text{ ampere.}$$

This is only an insignificant advantage over a single cell.

Suppose the external resistance had been 1 ohm. The current would then be

$$I = \frac{1.8}{1 + 0.25} = 1.44 \text{ amperes.}$$

This is about $1\frac{1}{2}$ times the current furnished by a single cell through the same resistance.

EXAMPLE. — Suppose now the cells be placed in series

through 1 ohm external resistance; what current will be obtained?

SOLUTION.— $I = \dfrac{4 \times 1.8}{1 + 4 \times 1} = 1.44$ amperes.

This is exactly what was given when the cells were all in parallel.

EXAMPLE.—Use the cells first in series, then in parallel upon $\frac{1}{4}$ ohm external resistance.

SOLUTION.—For series,

$$I = \frac{4 \times 1.8}{0.25 + 4} = 1.69 \text{ amperes.}$$

For parallel, $I = \dfrac{1.8}{0.25 + 0.25} = 3.6$ amperes.

The parallel arrangement gives more than twice the current given by the series connection.

Comparing the results of this problem with the previous problem having 30 ohms external resistance, it is apparent that for large external resistances relative to the battery resistance, the series method gives the larger current; while for relatively small external resistances, the parallel method may give the better results.

Multiple-Series Connection.—Sometimes, however, a better advantage is obtained by combining these two methods of connection in what is called the *multiple-series* method. The principles are the same as before; calling n the number in series and m the number in parallel, the formula for I becomes

$$I = \frac{nE}{R + \dfrac{nr}{m}}. \tag{41}$$

Example. — What current will be obtained through 30 ohms external resistance from 4 cells connected 2 in series and 2 in parallel, cell constants being 1.8 volts and 1 ohm? Draw diagram of connections.

Solution. — $I = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{2 \times 1.8}{30 + \dfrac{2 \times 1}{2}} = 0.116$ amp.

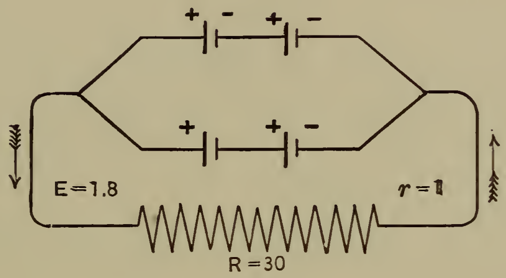


Fig. 8.

This current is about twice as much as when all were placed in parallel.

Example. — Suppose the external resistance had been 1 ohm, how much current would flow through it?

Solution. — $I = \dfrac{2 + 1.8}{1 + \dfrac{2 \times 1}{2}} = 1.8$ amperes.

This gives a larger current than either of the other methods of connecting the cells through the same resistance; both the series and the parallel gave 1.44 amperes.

The Best Arrangement. — The question naturally suggests itself, which of these methods for a given number of cells will give the most current through a given external resistance? As already suggested the matter is

roughly decided by a comparison of external and internal resistances. An observation of the numerical values given by the formulæ for these three cases, when the external resistance was 1 ohm, shows that in the *series* connection the internal resistance, 4 ohms, is larger than the external; in the *parallel* connection the internal, 0.25 ohm, is smaller than the external; in the *parallel-series* connection the internal is equal to the external. Hence the conclusion is that *the maximum current is obtained from a given number of cells connected to a given resistance when they are so arranged that the internal resistance is equal to the external.*

This is shown also from the following considerations. Since

$$I = \frac{nE}{R + \frac{nr}{m}} = \frac{E}{\frac{R}{n} + \frac{r}{m}},$$

it is apparent that I will be a maximum when $\frac{R}{n} + \frac{r}{m}$ is a minimum, for the numerator E is constant, being the E.M.F. of one cell. Now the product of $\frac{R}{n}$ and $\frac{r}{m} = \frac{Rr}{mn}$ is a constant, R and r both being constant, and mn the whole number of cells in the battery. It is a well-known principle and can be easily illustrated by a simple numerical example, that *the sum of two quantities is the least*, their product being a constant, *when the two quantities are equal.* Therefore $\frac{R}{n} + \frac{r}{m}$ will be a minimum when $\frac{R}{n} = \frac{r}{m}$; that is when $mR = nr$, and $R = \frac{nr}{m}$. But

$\frac{nr}{m}$ is the internal resistance of the battery. Therefore the *external and internal resistances must be equal for the greatest rate of flow.* But under these circumstances only one-half the total energy is expended in the external circuit, the other half being spent in the battery resistance. The efficiency of a battery arranged for greatest current is therefore 50%.

30. The Best Arrangement for a Required Current. — To find the number of cells for a required current it is only necessary to apply the general formula for the connection of cells,

$$I = \frac{nE}{R + \frac{nr}{m}},$$

remembering that for the greatest current the cells must be arranged so that $R = \frac{nr}{m}$.

Example. — How many cells arranged in series are necessary for $4\frac{4}{5}$ amperes through a resistance of $\frac{1}{2}$ ohm, the cell constants being 2 volts and $\frac{1}{4}$ ohm?

Solution. — Taking n = the number of cells in series and m = the number in parallel, we have

$$I = \frac{nE}{R + \frac{nr}{m}} = \frac{2n}{\frac{1}{2} + \frac{n}{4}} = 4\frac{4}{5} \quad (1).$$

$m = 1$, all being in series. Clearing (1)

$$2n = 2.4 + 1.2n;$$

and $$0.8n = 2.4; \quad n = 3 \text{ cells}.$$

Example. — How many cells in parallel will give 3.6 amperes through an external resistance of $\frac{1}{4}$ ohm, cell constants 1.8 volts and 1 ohm?

SOLUTION. — $I = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{1 \times 1.8}{\dfrac{1}{4} + \dfrac{1 \times 1}{m}} = 3.6$ amperes.

From this $1.8\,m - 0.9\,m = 3.6$; and $m = 4$ cells.

EXAMPLE. — How many cells and what arrangement will be necessary to supply 3 amperes, the external resistance being $1\frac{2}{3}$ ohms, and the cell constants 2 volts and 1 ohm?

SOLUTION. — $I = \dfrac{nE}{R + \dfrac{nr}{m}} = \dfrac{2\,n}{1\frac{2}{3} + \dfrac{n}{m}} = 3$ (1),

and $$2\,n = 5 + \frac{3\,n}{m} \quad (2).$$

Since $\dfrac{nr}{m} = R = 1\frac{2}{3}$, $m = \dfrac{n}{1\frac{2}{3}}$, since $r = 1$.

Substituting this value of m in (2),

$$2\,n = 5 + \frac{3\,n}{\dfrac{n}{1\frac{2}{3}}} = 5 + 5 = 10,$$

and $n = 5$ cells in *series*.

Also $m = \dfrac{n}{1\frac{2}{3}} = \dfrac{5}{1\frac{2}{3}} = 5 \times \frac{3}{5} = 3$ cells in *parallel.*

Hence it will require 15 cells, 5 series, 3 parallel.

For a required current at a given voltage the following rules may be applied with satisfaction.

Divide the cell E.M.F. by its resistance. (*a*) *If this short circuit current is twice or more than twice the required current, the cells will all be in series, and the number of cells will be*

$$n = \frac{E_1}{E - Ir}. \qquad (42)$$

E_1 is the required external E.M.F., E is the cell E.M.F., r the internal resistance, and I is the required current.

(*b*) *If the short circuit current is less than twice the required current and more than its equal, or less than equal and more than its half, or less than the half and more than its fourth, and so on, place as many cells in parallel as will make the short circuit current twice or more than twice the required current, then apply* (42) *for the number to be used in series.*

(*c*) *If the short circuit current is* 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, *etc., times the required current, place as many cells in series as will make the E.M.F. double the required line voltage, and then put enough in parallel to make the battery resistance equal to the external resistance.* — SLOANE.*

EXAMPLE. — There are 4 50-volt, 50-ohm lamps in parallel. Cell constants are 1.8 volts average and $\frac{1}{5}$ ohm. How many cells and what arrangement will supply the lamps?

SOLUTION. — Short circuit current $= \frac{1.8}{\frac{1}{5}} = 9$ amperes.

Required current $I = \frac{50}{50} \times 4 = 4$ amperes.

Hence, using (*a*),

$$n = \frac{E_1}{E - Ir} = \frac{50}{1.8 - 4 \times \frac{1}{5}} = 50 \text{ cells in series.}$$

EXAMPLE. — Using similar cells, how many will it take for 8 lamps similar to the above?

SOLUTION. — Short circuit current $= \frac{1.8}{\frac{1}{5}} = 9$ amperes.

Required current $I = \frac{50}{50} \times 8 = 8$ amperes.

* Sloane, *Arithmetic of Electricity*, p. 69 *et seq.*

Hence, using (*b*), *place* 2 *cells in parallel* giving

$$9 \times 2 = 18 \text{ amperes, cell current.}$$

Whence

$$n = \frac{E_1}{E - Ir} = \frac{50}{1.8 - 8 \times \frac{1}{2} \times \frac{1}{5}} = 50 \text{ cells in series.}$$

The whole number required is $mn = 2 \times 50 = 100$ cells arranged 2 in parallel and 50 in series.

EXAMPLE. — Suppose there were 18 lamps in parallel, how many cells would be necessary?

SOLUTION. — Short circuit current = 9 amperes.
Required current $I = \frac{50}{50} \times 18 = 18$ amperes.
Hence, using (*c*), we must place in series

$$n = \frac{2\,E_1}{E} = \frac{2 \times 50}{1.8} = 56 \text{ cells,}$$

giving an internal resistance of

$$\tfrac{1}{5} \times 56 = 11\tfrac{1}{5} \text{ ohms.}$$

Therefore we must place in parallel cells enough to reduce to $\frac{50}{18} = 2\frac{7}{9}$ ohms. Hence m in parallel equals $11\frac{1}{5} \div 2\frac{7}{9} = 4$ cells. Therefore there will be required $mn = 4 \times 56 = 224$ cells.

31. Arrangement of Cells for a Required Efficiency. — As for any generator of electrical energy, *the efficiency of a battery is the ratio of the external energy to the total energy developed*, and this is the same as *the ratio of the external to the total resistance in the circuit.*

$$\text{Eff.} = \frac{R}{R+r}. \tag{43}$$

In order to obtain a formula for r, multiply (43) by $R + r$, when $R \times \text{eff.} + r \times \text{eff.} = R$; or

$$r = \frac{R\,(1 - \text{eff.})}{\text{eff.}}. \tag{44}$$

Example. — What is the efficiency of a battery delivering current through an external resistance of 50 ohms when the battery resistance is 10 ohms?

Solution. — $\text{Eff.} = \frac{R}{R+r} = \frac{50}{50+10} = 83\frac{1}{3}\%$.

Example. — What must be the internal resistance of a battery when the external is 60 ohms in order that the efficiency will be $66\frac{2}{3}\%$?

Solution. — $r = \frac{R\,(1 - \text{eff.})}{\text{eff.}} = \frac{60\,(1 - .66\frac{2}{3})}{0.66\frac{2}{3}} = 30$ ohms.

Since $r = \frac{R\,(1 - \text{eff.})}{\text{eff.}}$ gives the internal resistance, the total resistance is expressed evidently by

$$\text{Total resistance} = R + \frac{R\,(1 - \text{eff.})}{\text{eff.}}.$$

If the required current is I, then the E.M.F. of the battery to give a certain current through a known resistance at a required efficiency will be, from Ohm's law,

$$E = \left[R + \frac{R\,(1 - \text{eff.})}{\text{eff.}}\right] I. \tag{45}$$

Example. — Required to provide storage cells for 4 50-volt, 50-ohm incandescent lamps in parallel. Cell constants 2 volts and $\frac{1}{10}$ ohm, the battery to work at 75% efficiency.

Solution.—

$$E=\left[R+\frac{R(1-\text{eff.})}{\text{eff.}}\right]I=\left[\frac{50}{4}+\frac{\frac{50}{4}(1-0.75)}{0.75}\right]\frac{50}{50}\times 4=66.6.$$

Hence the number of cells in series will be

$$n=\frac{66.6}{2}=34 \text{ cells.}$$

Also for 75% efficiency the cells must be arranged so that

$$r=\frac{R(1-\text{eff.})}{\text{eff.}}=\frac{\frac{50}{4}(1-0.75)}{0.75}=4.16 \text{ ohms.}$$

But the internal resistance of the 34 cells in series is $34 \times \frac{1}{10} = 3.4$ ohms, or already less than is required for 75%. Therefore 34 cells in series meet the requirements and the efficiency will be

$$\text{Eff.}=\frac{12.5}{12.5+3.4}=78\%.$$

32. Charging Storage Cells. — In charging batteries the E.M.F. applied must be equal to the ohmic drop in the wires and cells, plus the battery E.M.F. which acts as a counter electromotive force; it also must vary with the condition of charge in the cells, rising as the battery approaches full charge.

Example. — A battery of 45 cells is arranged in series for charging. Each has an internal resistance of 0.005 ohm and a counter E.M.F. of 2.1 volts. The leads have a resistance of 0.05 ohm. The cells take a charging current of 100 amperes; what must be the terminal voltage of the dynamo?

SOLUTION. —

$$\text{The ohmic drop} = (0.005 \times 45 + 0.05) \times 100$$
$$= 27.5 \text{ volts.}$$
$$\text{Counter E.M.F.} = 2.1 \times 45 = 94.5 \text{ volts.}$$

Hence the total voltage applied from dynamo switch-board must be

$$E = 94.5 + 27.5 = 122 \text{ volts.}$$

EXAMPLE. — Find the current at the beginning and end of the charge of 40 cells in series, each having 0.002 ohm internal resistance, leads 0.07 ohm, when the charge lasts 6 hours and the cell E.M.F. changes during charge from 2 volts to 2.5 volts. The generator remains uniformly at 125 volts.

SOLUTION. —

$$\text{Counter E.M.F. at first} = 40 \times 2 = 80 \text{ volts.}$$
$$\text{Counter E.M.F. at end} = 40 \times 2.5 = 100 \text{ volts.}$$
$$\text{Resistance of circuit} = (40 \times 0.002) + 0.07$$
$$= 0.15 \text{ ohm.}$$
$$\text{Ohmic drop at first} = 125 - 80 = 45 \text{ volts.}$$
$$\text{Ohmic drop at end} = 125 - 100 = 25 \text{ volts.}$$

Hence from Ohm's law,

$$I_1 = \frac{E_1}{R} = \frac{45}{0.15} = 300 \text{ amperes,}$$

and

$$I_2 = \frac{E_2}{R} = \frac{25}{0.15} = 166.66 \text{ amperes.}$$

33. The E.M.F. of Cells from the Available Heat of Chemical Action. — The source of the electrical energy of a cell is the chemical energy transformed in it. Neglect-

ing the losses which occur more or less in all cells, the electrical energy obtained is equivalent to the energy of chemical action. Now when elements combine or separate heat is liberated or absorbed. Therefore we have a measure of the chemical energy in the amount of heat exchanged. The amount of an element separated is also strictly proportional to the quantity of electricity flowing. These considerations furnish the principles for determining the E.M.F. necessary to separate certain chemical compounds, or produced by known chemical separations and combinations taking place in a cell.

The *atomic weight* of an element is the weight of its atom relative to that of hydrogen, H, which is 1.

The *valence* of an element is the number of hydrogen atoms to which it is equivalent in combining power, or the number of hydrogen atoms which would directly combine with or replace it. Thus water, H_2O, consists of two atoms of hydrogen held to one atom of oxygen. Hence O has a valence, or holding power, of 2.

The *chemical equivalent* of an element is its weight relative to hydrogen which enters into combination. In other words, it is the weight of the element which would combine with unit weight of hydrogen. It is obtained by dividing the *atomic weight* by the *valence*.

For example, in H_2O the atomic weight of O is 15.96; that of H is 1. But 15.96 of O is equivalent to $2 \times 1 = 2$ of hydrogen. Hence each unit of H is equivalent to $15.96 \div 2 = 7.98$ of O; that is to say, each gram of H will require for combination 7.98 grams of O. The *chemical equivalent* of oxygen is 7.98, or its atomic weight 15.96 divided by its valence 2.

The *electrochemical equivalent* of an element is the weight of it expressed as the fraction of a gram which will be separated by one coulomb of electricity. Clearly the electrochemical equivalent is proportional to the chemical equivalent. Thus 1 coulomb separates 0.00001038 gram of hydrogen, also 0.0000828 gram of oxygen, and these are in the same ratio as 1 to 7.98, or as the chemical equivalents. If we take the reciprocal of the electrochemical equivalent we obtain the quantity of electricity necessary to separate 1 gram of the element. See table of chemical constants.

Example. — Obtain the formula to be used in obtaining the electromotive force in volts produced in any cell whose *thermal equivalents* are known; that is when the calories of heat formed or absorbed in the chemical reactions can be obtained.

Solution. — The electrical energy developed in any cell is

$$\text{Energy} = EIt.$$

The available heat energy for transformation into electrical energy is

$$\text{Heat} = JH,$$

in which H is the free heat in calories due to chemical action, and J is Joule's Equivalent, or the *ergs* of energy represented by each unit of heat $= 4.19 \times 10^7$. If expressed in ergs the electrical energy will be

$$E_e = EIt \times 10^7 \text{ ergs}.$$

Therefore

$$JH = EIt \times 10^7 = 4.19 \times 10^7 \times H.$$

5. Chemical and Electrochemical Constants.

Names.	Symbol.	Valence.	Atomic Weight.	Chem. Equiv.	Electro-chemical Equivalent, in Grams per Coulomb.	1 Electrochemical equivalent, or Coulombs per Gr.
Electropositive El.						
Hydrogen . .	H	1	1.00	1.00	0.00001038	96,340.0
Potassium . .	K	1	39.03	39.03	0.0004051	2,469.0
Sodium . . .	Na	1	23.00	23.00	0.0002387	4,189.0
Silver	Ag	1	107.70	107.70	0.001118	894.5
Copper (ic) . .	Cu	2	63.18	31.59	0.0003279	3,050.0
Mercury (ic) .	Hg	2	199.80	99.90	0.001037	964.3
Tin (ic) . . .	Sn	4	117.40	29.35	0.0003046	3,283.0
Iron (ic) . . .	Fe	3	55.88	18.63	0.0001933	5,171.0
Nickel (ic) . .	Ni	2	58.60	29.30	0.0003041	3,287.0
Lead	Pb	2	206.40	103.20	0.001071	933.7
Zinc	Zn	2	64.88	32.44	0.0003367	2,970.0
Electronegative El.						
Oxygen . . .	O	2	15.96	7.98	0.00008283	12,070.0
Chlorine . . .	Cl	1	35.37	35.37	0.000367	2,724.0
Nitrogen . .	N	3	14.01	4.67	0.00004847	20,630.0

From which $EIt = 4.19\ H.$

But It, the number of coulombs corresponding to 1 gram of hydrogen, or 1 chemical equivalent of any other element, as 107.7 grams of silver, 31.59 grams of copper, etc., is 96,340 coulombs; see table 5. Hence, since

$$It = 96,340,$$

$$E \times 96,340 = 4.19\ H$$

or
$$E = 0.000043\ H.^* \qquad (46)$$

If H is expressed in *kilogram-degree* calories, instead of *gram-degree* calories, (46) becomes

$$E = 0.043\ H. \qquad (47)$$

* Exactly, $E = 0.000043\ H + \kappa T$, where κ is the temperature coefficient of E.M.F. of the cell, and T is the absolute temperature.

We thus obtain the electromotive force from the resultant heat of chemical action, keeping in mind that the result can only be approximate because of local action and heat absorbed from or given up to surroundings. The heats of combination of many *ions*, that is atoms or groups of atoms, have been determined. See table 6 for the heats resulting from the chemical reactions taking place in the most common cells. It is to be observed that H in the formulæ is the algebraic heat; that is, the difference between the heats of formation and those of disintegration in the cell.

Example. — What is the minimum voltage a cell may possess to decompose water, H_2O?

Solution. — 1 gram H combining with 7.98 grams O liberates 36,500 calories; see table 6. Hence to separate them an equivalent amount of energy is necessary. Therefore

$$E = 0.000043 \times 36{,}500 = 1.56 \text{ volts.}$$

Example. — What is the approximate E.M.F. of the Daniell cell?

Solution. — In this cell the elements are

Zinc | sulphuric acid —— copper sulphate | copper.

The chemical reactions are

$$Zn + H_2SO_4 = ZnSO_4 + H_2. \qquad (1)$$
$$CuSO_4 + H_2 = H_2SO_4 + Cu. \qquad (2)$$

$$(1)\begin{cases} \text{Heat liberated by the formation of } ZnSO_4 = 53{,}500 \\ \text{Heat absorbed by the separation of } H_2SO_4 = \underline{36{,}500} \\ \text{Heat available for transformation} = 17{,}000 \end{cases}$$

(Calories.)

		CALORIES.
(2)	Heat liberated by the formation of H_2SO_4	= 36,500
	Heat absorbed by the separation of $CuSO_4$	= 28,400
	Available for transformation	= 8,100

Total available heat = 17,000 + 8,100 = 25,100 calories.

Therefore $E = 0.000043 \times 25{,}100 = 1.0793$ volts.

6. HEATS OF FORMATION OR SEPARATION.

COMPOUNDS.	IONS.	CALORIES HEAT.
H_2SO_4	H_2 and SO_4	36,500
$ZnSO_4$	Zn and SO_4	53,500
H_2O	H_2 and O	36,500
HCl	H and Cl	19,650
$ZnCl_2$	Zn and Cl_2	56,400
$PbCl_2$	Pb and Cl_2	39,200
CuO	Cu and O	20,200
$Zn(OH)_2$	Zn and 2 (OH)	41,800
$Zn(OH)_2 + 2\,KOH$	$Zn(OK)_2$ and 2 H_2O	8,000
$PbSO_4$	Pb and SO_4	37,400
AgCl	Ag and Cl	14,600
PbO	Pb and O	25,500
PbO_2	Pb and O_2	31,570
CO_2	C and O_2	51,300
$CuSO_4$	Cu and SO_4	28,400

34. **Material Consumed in a Battery.** — It is sometimes interesting, if not essential, to compute the amount of chemicals used in a cell or a battery for a given amount of work done by the cell. For this purpose the following may be used with fair accuracy.

The electrochemical equivalent in grams per coulomb multiplied by the number of molecules of the compound, or the number of atoms of the element going into combination, will give the *number of grams consumed per coulomb,*

or *per ampere-second.* If this result be divided by the voltage of the cell, *the result will be in grams per watt.* This now multiplied by 3600 will give the *number of grams used per watt-hour.*

Hence
$$w = \frac{Z \times N}{E} \times 3600. \tag{48}$$

In this w is the weight in grams per watt-hour; Z is the electrochemical equivalent; N is the number of molecules or atoms entering combination and is obtained from the chemical equation; E is the cell E.M.F.

EXAMPLE. — How much zinc is used up in a Daniell cell working on a total resistance of 10 ohms for 1 hour?

SOLUTION. — For zinc, $Z = 0.0003367$; $E = 1.08$ volts. $I = \frac{1.08}{10} = 0.108$ ampere. The chemical equation is

$$Zn + H_2SO_4 = ZnSO_4 + H_2.$$

Hence there is one atom of Zn entering into combination. Hence

$$w = \frac{0.0003367 \times 1}{1.08} \times 3600 = 1.12 \text{ grams per watt-hour.}$$

$$\text{Total watt-hours} = 1.08 \times 0.108 \times 1 = E \times I \times \text{hrs.}$$
$$= 0.11664.$$

Therefore total zinc dissolved in 1 hour is

$$W = 1.12 \times 0.11664 = 0.1306 \text{ grams.}$$

The work may be slightly simplified by using a formula for total weight at once, obtained by multiplying the

formula for grams per watt-hour by the formula for the number of watt-hours.

$$W = \frac{Z \times N \times 3600}{E} \times EI \times \text{hours} = 3600\, ZNI \times \text{hours},$$

or
$$W = 3600\, ZNIT, \tag{49}$$

in which W is the total weight separated or combined, N is the number of atoms or molecules combining, I is the current in amperes, and T is the time in hours during which the current flows. If E and R are known $\frac{E}{R}$ may be put in place of I. The formula may be still further shortened by getting Z, the electrochemical equivalent, in grams per *coulomb-hour*, then (49) becomes

$$W = ZNIT. \tag{50}$$

EXAMPLE. — How much $CuSO_4$ is used up under the conditions named in the last problem?

SOLUTION. — The chemical equation is

$$CuSO_4 + H_2 = H_2SO_4 + Cu.$$

Hence 1 molecule of $CuSO_4$ is separated, and $N = 1$. For copper, Cu, at. wt. = 63.18. For (SO_4) at. wt. = $32 + (15.98)4 = 95.92$. Total molecular weight $CuSO_4$ = $63.18 + 95.92 = 159.1$. Z for Cu = 1.18 grams per coulomb-hour. For $CuSO_4$,

$$Z = 1.18 \times \frac{159.1}{63.18} = 2.9 \text{ grams per coulomb-hour.}$$

Therefore from (50)

$$W = ZNIT = 2.9 \times 1 \times 0.108 \times 1 = 0.313 \text{ gram.}$$

35. Original Problems. — 1. What is the greatest external resistance that may be used when the battery constants are 5 volts and 2 ohms, in order that $1\frac{1}{2}$ amperes may be obtained? $R = 1.33$ ohms.

2. It was observed that a cell whose electromotive force is 1.8 volts gave 1 ampere through an external resistance of 1 ohm; what was the internal resistance of the cell? $r = 0.8$ ohm.

3. What is the E.M.F. of a cell whose internal resistance is 1 ohm, when 1 ampere flows through an external resistance of 1 ohm? $E = 2$ volts.

4. The cell constants are 2 volts and $\frac{1}{4}$ ohm. Required 5 amperes through an external resistance of 1.5 ohms; how many cells joined in series will be required?
$n = 10$ cells.

5. Can 5 amperes be obtained through the same resistance with the cells joined in parallel? Show why.

6. What must be the internal resistance of each cell and of the whole battery when 5 cells joined in parallel through an external resistance of 0.835 ohm gives 2 amperes, E being 1.75 volts? Each cell $= \frac{1}{5}$ ohm.
Battery $= \frac{1}{25}$ ohm.

7. Which is the better arrangement of 4 cells whose constants are two volts and $\frac{1}{2}$ ohm, series or parallel, when the external resistance is 20 ohms? Is there any better arrangement of 4 cells? (*a*) Series. (*b*) No.

8. What is the largest external resistance that can be used in 7, in order that the parallel connection will give a current equal to the series connection?
$R = 0.5$ ohm.

9. What current will be obtained under the conditions of problem 8 when two of the cells are placed in series and 2 in parallel? $I = 4$ amperes.

10. Find the number of these cells required in parallel when 5 are placed in series to supply small lamps of 5 ohms resistance with 1.8 amperes of current.

$m = 5$ cells in parallel.

11. Find the whole number of cells to be used under the following conditions: cell constants 2 volts and $\frac{1}{5}$ ohm external resistance 10 ohms; when n are put in series and m in parallel the current is 0.97 ampere; but when m are put in series and n in parallel only 0.78 ampere is obtained. $mn = 4 \times 5 = 20$ cells.

12. Group the cells in problem 11 so as to derive the greatest possible current under the conditions given. Determine the efficiency in 11 and 12.

Place 20 in series.
Eff. 11 = 97.5%.
Eff. 12 = 71.4%.

13. What is the least number of cells and their arrangement for 2 amperes through an external resistance of 1.5 ohms, cell constants 2 volts and $\frac{1}{2}$ ohm?

$n = 3$ cells in series.
$m = 1$ cell in parallel.

14. Ninety cells whose internal resistance is each 1 ohm are available for a circuit whose resistance is 10 ohms; arrange the cells for the greatest current.

$n = 30$ cells in series.
$m = 3$ cells in parallel.

15. What is the least number and the arrangement of cells of 2 volts 4 ohms each, when 1 ampere is desired in an external resistance of 15 ohms?

$n = 15$ cells; $m = 4$ cells.

16. There are 4 110-volt, 220-ohm lamps in parallel; storage cells are at hand of 2 volts and $\frac{1}{5}$ ohm each. How many cells will it require to give the necessary current?

$n = 69$ cells in series.

17. Suppose the cell constants were 2 volts and 1 ohm; what number and arrangement would be required in problem 16? $n = 110$; $m = 2$; total 220.

18. If the E.M.F. required in the circuit be 50 volts and the current 5 amperes, cell constants 2 volts and $\frac{1}{4}$ ohm, how many cells will be required?

$n = 37$; $m = 2$; total 74 cells.

19. Solve problem 15 for 60% efficiency.

$n = 13$; $m = 5$.

20. How many cells would be required in problem 16 to work at 75% efficiency? $n = 74$ cells.

21. Solve problem 17 for 80% efficiency.

$n = 69$; $m = 5$; total 345.

22. What is the least voltage a cell may have to be used for decomposing hydrochloric acid, HCl, when platinum electrodes are used? $E = 0.84$ volt.

23. Zinc and platinum are used as electrodes in simple HCl. What will be the maximum E.M.F. of the cell?

$E = 0.7353$ volt.

24. Suppose copper sulphate, $CuSO_4$ be used in the last cell in a porous cup with a platinum, Pt, electrode. What will be the cell E.M.F.?

$E = 1.0836$ volt.

25. The final action in the "silver chloride" cell is represented by the equation

$$2\,AgCl + Zn = ZnCl_2 + 2\,Ag.$$

What is its E.M.F.? $E = 1.17$ volts.

26. How much zinc, Zn, will be used in 10 hours when 10 Daniell cells are connected 5 in series and 2 in parallel through a resistance of 5 ohms, cell constants being about 1.08 volts and 2 ohms?

Zn used = 6.54 grams.

27. How much $CuSO_4$ will be used in the above arrangement? $CuSO_4$ used = 15.66 grams.

28. The final action in the Becquerel cell is represented by the equation, $PbSO_4 + Zn = ZnSO_4 + Pb$.

Determine the E.M.F. and the amount of Pb separated in a cell working 10 hours on 1 ohm resistanee?

$E = 0.69$ volts.

29. Find the E.M.F. of the Edison-Lelande cell whose chemical formulæ are as follows:

(1) $Zn + 2H_2O = Zn(OH)_2 + H_2.$

(2) $\begin{cases} Zn(OH)_2 + 2KOH = Zn(OK)_2 + 2H_2O. \\ H_2 + CuO = H_2O + Cu. \end{cases}$

SOLUTION. — In the first and second reactions 41,800 + 8000 calories are set free. In the last reaction 20,200 calories are absorbed. Hence the available heat is 49,800 − 20,200 = 29,600 calories. Hence

$$E = 29600 \times 0.000043 = 1.27 \text{ volt.}$$

30. Twenty small storage cells are to be charged with a minimum of 10 amperes. If the highest cell E.M.F. is 2.5 volts, cell resistance 0.008 ohm, and wire resistance 0.14 ohm, with what voltage should the cells be charged? If a 110-volt incandescent machine is the only one available, how can arrangements be made to charge the cells by means of it?

(*a*) Charging E.M.F. = 53 volts.

(*b*) Place 10 50-volt lamps in parallel, and in series with the cells to be charged; then reduce the machine voltage by means of its field rheostat till 10 amperes are obtained.

31. There are 50 storage cells whose constants may be taken at 2.1 volts and 0.002 ohm to be charged from a 250-volt circuit. The connecting wires, leads, etc., have about 0.9 ohm resistance; how much additional resistance must be added in series so that the cells may be charged with about 40 amperes of current?

$R = 2.6$ ohms.

32. How many storage cells may be charged in series on a 110-volt circuit when 50 amperes are required, the cell E.M.F. being about 2 volts, resistance 0.004 ohm, leads and connecting wires 0.088 ohm?

Number in series = 48 cells.

VII.

MAGNETISM.

36. **Units and Definitions.** — (a) **Magnetic flux** is analogous to current flow in electrical units. In a magnetic field the flux originates at a north pole and passes through space into a south magnetic pole, thence through the magnet to the north pole, thus completing the *magnetic circuit.* If we conceive a unit pole as previously defined to be placed at the center of a spherical surface whose radius is 1 centimeter, then the whole flux passing through this spherical surface will be 4π lines of force, or 4π *maxwells;* for unit pole will produce unit intensity, or one line per square centimeter, at unit distance, and there are $4\pi r^2$ units of surface in a sphere; in this case $4\pi r^2 = 4\pi$ square centimeters, hence 4π lines total flux.

The total magnetic flux is represented by the Greek letter ϕ, and is expressed in *maxwells.* Thus a magnetic pole sends out 500,000 lines of force; the flux is

$$\phi = 500{,}000 \text{ maxwells.}$$

(b) The **density** of magnetic flux, or the *density* of the magnetization, or simply, the magnetic *induction* is the number of lines per square centimeter or per square inch, and is represented by B; it is expressed in *gausses* when the area is in square centimeters. For example, suppose the magnet carrying the 500,000 maxwells of flux

have 100 square centimeters across sectional area, then the intensity of magnetization is

$$B = \frac{500{,}000}{100} = 5000 \text{ gausses.}$$

In general $$B = \frac{\phi}{A}, \text{ or } \phi = BA.$$

(c) **The intensity of field, or strength of field,** is the number of gausses in air, and is denoted by the symbol H. Thus the flux in 100 square centimeters of air is $\phi = 2000$ maxwells; the intensity of field is $H = \frac{2000}{100} = 20$ gausses. The intensity of induction in a bar of iron placed in this field is $B = H\mu$, in which μ is the permeability of the iron, or the ratio of its intensity to that of the field. Let the permeability of a certain piece of iron be 250. Then

$$B = 20 \times 250 = 5000 \text{ gausses.}$$

(d) The **energy** due to magnetization varies as the square of the field density, or as H^2. This is analogous to mechanical energy which varies as the square of velocity, V^2. This relation only holds good for air free from any metallic influences. Take a field density of 200 gausses; then the energy in each cubic centimeter of space is, from Maxwell,*

$$\text{Energy} = \frac{H^2}{8\pi}. \qquad (51)$$

Putting in known quantities we get

$$\text{Energy per c.c.} = \frac{(200)^2}{8 \times 3.1416} = 1592 \text{ ergs.}$$

* Maxwell, *A Treatise on Electricity and Magnetism*, Vol. II. Art. 633.

In 10 cc. the energy would be

$$1592 \times 10 = 15{,}920 \text{ ergs} = \frac{15{,}920}{10^7} = 0.001592 \text{ joule.}$$

(e) **Magnetic reluctance** is represented by $\mathcal{R}$, and the unit is the *oersted.* It corresponds exactly to electrical resistance, and is the force opposing the magnetic flux. The *reluctivity*, or *specific reluctance*, corresponds to specific resistance, and is the reluctance of 1 cc. of the material forming any part of the magnetic path, or circuit. Hence the total reluctance in *oersteds* is

$$\mathcal{R} = k\frac{l}{a}. \tag{52}$$

Here k is the reluctivity, l the length of the portion considered, a the cross section. The reluctivity of vacuum is unity. This is practically true of air, wood, copper, glass, paper, and in fact all non-magnetic substances. For iron and other magnetic substances it is less than 1 but varies with B. The reciprocal of reluctivity is *permeability*, and is expressed by μ, as previously explained.

$$\mu = \frac{1}{k}.$$

(f) **Magnetomotive force** is analogous to electromotive force. It is the force setting up the magnetism or magnetic flux, and is represented by M.M.F., or simply by M. It is expressed in *gilberts.* Suppose the flux required is $\phi = 500{,}000$ maxwells, and the magnetic reluctance of the circuit is 5 *oersteds.* The M.M.F. would then be $M = \phi\mathcal{R} = 500{,}000 \times 5 = 25 \times 10^5$ *gilberts.* While the absolute unit is the gilbert, the practical value for electromag-

netism is given in *ampere-turns*, or the number of amperes multiplied by the number of turns in the coil $= nI$. But $M = 1.256nI$, so that the absolute value is easily obtained from the ampere-turns. *Gilberts* $= 1.256 \times$ *ampere-turns.*

(g) The **magnetizing force** in air is the intensity of the field H. When applied to a solenoid or electromagnet it means the M.M.F. per centimeter of length of the coil expressed in gilberts, and is still represented by H. Thus a coil of wire 10 centimeters long develops an M.M.F. of 5000 gilberts. Here the magnetizing force is $H = \frac{5000}{10}$ $= 500$ gilberts per centimeter; but since the reluctance of 1 c.c. of air is 1, 500 is also the flux per square centimeter, or H. This is the M.M.F. which sets up the magnetism and maintains it in 1 centimeter of the coil's length.

In general

$$H = \frac{M}{l}. \tag{53}$$

(h) By **magnetic leakage** is meant that portion of the flux which passes through such paths as to be unavailable for the purposes for which the electromagnet has been constructed. Thus in a dynamo the leakage lines are those passing around the armature, or across from leg to leg, so as not to be cut by the wires of the armature. If ϕ_f is the flux in the field cores, and ϕ_a is that passing through the armature core, the *coefficient* of *leakage* is

$$v = \frac{\phi_f}{\phi_a}. \tag{54}$$

In dynamos v varies from 1.1 to 1.7. Say for average machines, $v = 1.35$.

(i) A magnet is **saturated** when it has become so strongly magnetized, and its permeability is therefore so reduced that further magnetomotive force does not appreciably increase the flux. As the current is made to increase in the winding of an electromagnet the flux rapidly increases at first, its permeability decreasing as the intensity of magnetism increases; the increase in flux becomes less rapid as the magnet approaches saturation, until finally further increase of current fails to produce any greater intensity. If now the current decrease, the magnetism falls though less rapidly than the current to a certain point below which it does not go even when the current becomes zero. The amount of magnetism thus remaining is called *remanent* or *residual magnetism*.

(j) **Magnetic hysteresis** is the frictional resistance to the turning around of the iron molecules which takes place during magnetization or reversals of magnetization. This explains why the change in the magnetism *lags* behind the change in the magnetizing current as stated above. Hysteresis is especially to be noted in iron subjected to rapidly alternating magnetizing forces, as in armatures of dynamos and in alternating current transformers. It varies with the frequency and with the 1.6th power of the intensity of induction. Steinmetz * has expressed the hysteretic loss for transformers, obtained from experiment, by a formula similar to the following:

$$W_h = hfB^{1.6}, \tag{55}$$

in which W_h is the loss in watts per cubic centimeter due to hysteresis, h is the hysteretic constant which

* Compare Steinmetz, *Alternating Current Phenomena*, Art. 98.

varies with the quality of iron and steel from 20×10^{-11} to 10×10^{-9}. Steinmetz found certain sheet iron transformer plates to have $h = 24 \times 10^{-11}$. Possibly a good average for transformer plates would be 21×10^{-11}. f in the formula is the frequency of the alternating current and B is the intensity of induction in gausses.

(k) **Eddy** or **Foucault** currents should properly be considered here, since the effect is observed in iron cores. They are irregular currents developed in iron subjected to varying magnetic induction. Like hysteresis their energy is wasted in heating the iron. Steinmetz * has developed a formula for calculating the losses due to eddy currents in laminated iron, which may be stated thus:

$$W_e = (tfB)^2 \times 10^{-16}. \tag{56}$$

W_e is the Foucault loss per cubic centimeter, t the thickness in mils of the laminations, or plates, f the frequency, and B the maximum induction as in the hysteresis formula.

The total loss in V cubic centimeters is therefore

$$W_e = V(tfB)^2 \times 10^{-16}. \tag{57}$$

To express in watts per pound of iron we have

$$W_e = 6\,V(tfB)^2 \times 10^{-15}. \tag{58}$$

V is the volume in cubic inches. The eddy currrent loss is only 15% to 25% of the total core losses.

* Reference cited, Art. 89.

VIII.

RELATION OF MAGNETIC QUANTITIES.

37. **The Law of Magnetic Force.** — The definition already given for *unit pole* is one which will exert upon a similar one at a distance of 1 centimeter a force of 1 dyne.

EXAMPLE. — Develop a formula for the force exerted between two magnetic poles.

SOLUTION. — Let m and m' be the pole strengths and d the distance between them. First take the distance 1 cm. Then if one pole had unit strength and the other strength m, the force, from the definition would be $F = m \times 1 = m$ dynes. But the pole strength is m' instead of unity. Hence the force at 1 cm. will be $m \times m' = mm'$ dynes of attraction or repulsion, depending upon whether they are unlike or like poles. Now since the magnetic field extends in all directions, the *surface* of influence will increase as the square of the distance away from the pole; therefore the *intensity* of the force will decrease in the same ratio. If the distance is d, the force will be $\frac{1}{d^2}$th of mm'; or

$$F = \frac{mm'}{d^2}. \tag{59}$$

This law may be stated thus in words: *Magnets attract or repel each other with a force proportional to the product of*

their pole strengths and inversely proportional to the square of their distance apart.

The forces due to a magnet pole at different distances may readily be determined by using a short piece of magnetized knitting needle suspended on a silk thread, obtaining the number of its swings in a given time at the different distances from the pole to be tested. It can readily be shown that the strength of field at any point is proportional to the square of the number of oscillations made by the needle at this point. Different pole strengths can be compared in the same way by taking the swings at the same distance from each. Of course the earth's field must be taken into account in these tests. The following examples will illustrate these points.

Example. — Compare the earth's field intensity at a point A where the needle makes 24 vibrations a minute with its intensity at B where it makes 36 vibrations per minute.

Solution. — Calling the horizontal intensity of the earth's field at the two places respectively H_a and H_b, from the principle stated we have

$$\frac{H_a}{H_b} = \frac{N_a^2}{N_b^2}. \qquad (60)$$

$\frac{N_a}{N_b}$ is the ratio of the number of vibrations at A and B respectively. Hence $\frac{H_a}{H_b} = \frac{\overline{24}^2}{\overline{36}^2} = \frac{4}{9}.$

Example. — It is found that at the point A the needle makes 60 swings per minute when suspended 20 cm. from

a certain long magnet which we shall call 1; it also makes 72 swings per minute when placed the same distance from long magnet 2. Compare the intensities of the two magnet fields.

Solution. — The forces in the two cases causing the vibrations are due to two things: the earth's field and the magnet field, since the needle was so arranged with respect to the magnet and the earth as to be in the magnetic meridian passing longitudinally through the magnet. Let H represent the earth's field intensity, and M_1 and M_2 respectively the magnet fields together with the earth's field. Also let N be the number of swings in the earth's field alone, and N_1 and N_2 be the numbers due to the total field in the two cases. Hence

$$\frac{F_1}{F_2} = \frac{M_1 - H}{M_2 - H} = \frac{N_1^2 - N^2}{N_2^2 - N^2}. \tag{61}$$

Here F_1 and F_2 represent the forces due to the magnets alone. For the particular values given, therefore,

$$\frac{F_1}{F_2} = \frac{\overline{60}^2 - \overline{24}^2}{\overline{72}^2 - \overline{24}^2} = \frac{21}{32}.$$

The intensity of field due to the first magnet pole at 20 c.m. is $\frac{21}{32}$ of that due to the second magnet pole at the same distance.

Example. — At what distance apart are two magnet poles when a mutual force of 500 dynes is measured between them, the poles having 100 and 500 units of strength respectively?

SOLUTION. —

$$F = \frac{mm'}{d^2} = \frac{100 \times 500}{d^2} = 500.$$

Hence

$$d = \sqrt{\frac{100 \times 500}{500}} = 10 \text{ centimeters.}$$

EXAMPLE. — Two poles of equal strength are 20 centimeters apart and the force of mutual attraction is 2.25 dynes. Find the strength of the poles.

SOLUTION. — From the above formula

$$mm' = Fd^2 = 2.25 \times \overline{20}^2 = 900.$$

But $m = m'$; hence $m^2 = 900$, and $m = 30$. Also $m' = 30$.

EXAMPLE. — Show that the following experimental data prove that the force due to a magnet varies inversely as the square of the distance from the pole. The small magnetic needle used made 15 swings in a minute in the earth's field alone, and when suspended in the meridian of the earth and a bar magnet at a distance of 20 centimeters from the latter it made 25 swings in a minute; when placed 40 centimeters from the magnet pole it made 18 swings in a minute.

SOLUTION. —

$$\frac{F_1}{F_2} = \frac{N_1^2 - N^2}{N_2^2 - N^2} = \frac{\overline{25}^2 - \overline{15}^2}{\overline{18}^2 - \overline{15}^2} = \frac{4}{1}.$$

The force at 20 cm. is four times the force at 40 cm. while the distance is $\frac{1}{2}$. Therefore

$$\frac{F_1}{F_2} = \frac{d_2^2}{d_1^2} = \frac{\overline{40}^2}{\overline{20}^2} = 4,$$

or the forces are inversely as the squares of the distances.

EXAMPLE. — The vibrating needle swinging in the earth's field makes 10 vibrations a minute; the *n*-pole of a magnet which is placed in the magnetic meridian is brought near the *s*-pole of the needle when the latter makes 20 vibrations a minute. How much stronger field does the magnet produce at the given distance than the earth?

SOLUTION. —

$$\frac{F_1}{F_2} = \frac{N_1^2 - N^2}{N^2} = \frac{\overline{20}^2 - \overline{10}^2}{10^2} = 3.$$

The magnet's field is 3 times as intense as the earth's field.

38. Magnetic Flux, Intensity of Magnetism and Strength of Field. — Conceive a spherical surface, radius one centimeter, drawn about a pole whose strength is 150 units; how many maxwells of flux will pass through the surface?

SOLUTION. — The surface is 4π sq. cm., and since the pole is 150, the density on the surface will be 150 lines per sq. cm. Hence

$$\phi = 4\pi \times 150 = 1885 \text{ maxwells.}$$

EXAMPLE. — What must be the intensity of magnetism when a magnet of 20 sq. cm. cross section is excited to produce 200,000 maxwells? Find μ from the table of permeabilities.

SOLUTION. — From the formula given

$$B = \frac{\phi}{A} = \frac{200{,}000}{20} = 10{,}000 \text{ gausses.}$$

Hence μ for annealed iron $= 2000$.

EXAMPLE. — What is the intensity of a magnetic field when a bar of soft iron which is placed in it has an induced intensity of 16000 gausses at a permeability of 300?

SOLUTION. — Since $B = \mu H$,

$$H = \frac{B}{\mu} = \frac{16{,}000}{300} = 53.3 \text{ gausses.}$$

EXAMPLE. — What is the permeability of a piece of soft iron made into a ring so that a field of 100 gausses produces an induction, or intensity of magnetism, of 18000 gausses?

SOLUTION. — From $B = \mu H$,

$$\mu = \frac{B}{H} = \frac{18{,}000}{100} = 180.$$

The relation of B and H may be experimentally determined by what is known as the *permeameter method.** A rectangular piece of iron has a slot cut through it in which the magnetizing coil is placed. The coil has a cylindrical hole through it longitudinallly to receive the rod to be tested. The iron is drilled above the coil to permit the rod's being placed in the coil. The lower end of the rod is surfaced to make good contact with the iron below the coil. When the current is turned on the attractive force causes the rod to stick so that a force is necessary to pull it out, which is measured by a spring balance.

Suppose the coil to be 12 cm. long and have 300 turns,

* See Thompson, *Lectures on the Electromagnet*, p. 70.

thus making 25 turns to the centimeter. The intensity of field, or magnetizing force, is then, per ampere of current,

$$H = 1.25 \times nI = 1.25 \times 25 \times 1 = 31.25 \text{ gausses.}$$

If 10 amperes be used,

$$H = 31.25 \times 10 = 312.5 \text{ gausses.}$$

Now suppose the area of contact, or end of the rod, be A sq. in., and the pull to detach it be W pounds. Then the formula for B, whose derivation is given below, is

$$B = 1317\sqrt{\frac{W}{A}} + H. \tag{62}$$

EXAMPLE. — If A is in square centimeters and W is in grams, what will the formula become?

SOLUTION. — Since 1 cm. $= \frac{2}{5}$ in. and 1 in $= \frac{5}{2}$ cm., 1 sq. in. is then $\frac{25}{4}$ sq. cm.; also, since 1 lb. = 453.6 grams, the substitution of the proper constants gives

$$B = 1317\sqrt{\frac{W \times 453.6}{A \times \frac{25}{4}}} + H.$$

Taking the square root of the constants under the radical sign and placing the result outside, this reduces to

$$B = 156\sqrt{\frac{W}{A}} + H. \tag{63}$$

This is to be used when A is sq. cm., and W is grams to pull the rod out.

EXAMPLE. — To derive this last formula from Maxwell's formula for the lifting power of magnets; namely,

$$W = \frac{B^2 A}{981 \times 8\pi}, \text{ as explained later.}$$

SOLUTION. — In this W is the number of grams of weight lifted, A is the number of square centimeters of contact of the magnet pole with the armature or keeper. Transposing this,

$$B^2 = \frac{W \times 981 \times 8\pi}{A}.$$

Whence
$$B = \sqrt{981 \times 8\pi} \times \sqrt{\frac{W}{A}} = 156\sqrt{\frac{W}{A}}.$$

When the induction is caused by a field of H gausses, this becomes
$$B = 156\sqrt{\frac{W}{A}} + H.$$

EXAMPLE. — Find the permeability of a sample of charcoal iron when a test by the permeameter described gave the following data: coil carries 20 amperes, area of rod tested is 1 square centimeter, and it requires a force of 5794.25 grams to pull the rod out.

SOLUTION. — First obtain B.

$$B = 156\sqrt{\frac{5794.25}{1}} + 31.25 \times 20 = 12{,}500 \text{ gausses.}$$

Therefore
$$\mu = \frac{B}{H} = \frac{12{,}500}{625} = 20.$$

39. Ohm's Law and the Magnetic Circuit. — As the magnetic circuit is analogous to the electric circuit, so the law governing the magnetic flux is analogous to Ohm's law governing electric currents. Expressing the amount of flux by ϕ, the magnetomotive force by M, the reluctance by $\mathcal{R}$, we have
$$\phi = \frac{M}{\mathcal{R}}. \tag{64}$$

EXAMPLE. — How many oersteds of reluctance will there be in a wooden ring whose mean circumference is 100 centimeters and whose cross section is 20 square centimeters?

SOLUTION. — Since $\mathcal{R} = k\frac{l}{a}$, and in wood $k = 1$,

$$\mathcal{R} = 1 \times \frac{100}{20} = 5 \text{ oersteds.}$$

EXAMPLE. — Suppose the wooden ring wound uniformly with 1000 turns of insulated wire and 10 amperes of current sent through it; what will be the M.M.F. in gilberts and also the magnetizing force, or field intensity?

SOLUTION. — $nI = 1000 \times 10 = 10{,}000$ ampere-turns.

Also $M = 1.256 \times nI = 1.256 \times 10^4 = 12{,}560$ gilberts.

$$H = \frac{M}{l} = 125.6 \text{ gilberts per cm. or gausses.}$$

EXAMPLE. — Show how $M = \frac{4\pi nI}{10} = 1.256nI$.

SOLUTION. — Assume unit pole within a long helix having n turns to the centimeter and carrying I amperes. Now let the pole be moved along the axis of the coil 1 centimeter, when each of the 4π lines of force from the unit pole will be intercepted by n turns of wire, or the lines cut will be $4\pi n$; and since there are I amperes the work done in moving the pole is $4\pi nI$, in which I is in absolute amperes. But work is Fl, or in this case Hl, where H is the magnetizing force. Hence

$$Hl = 4\pi nI;$$

but $l = 1$, therefore $H = 4\pi nI$, in this particular instance. If, however, the pole had been moved the whole length l of the solenoid, the whole work done is

$$Hl = 4\pi nI = M, \tag{65}$$

since $M = Hl$, or the gilberts-per-centimeter times the number of centimeters in the coil. Now if I is in international amperes,

$$M = \frac{4\pi nI}{10} = 1.256\, nI.$$

EXAMPLE. — How many maxwells of flux will pass around the ring above described under the conditions given? What is the intensity of magnetization?

SOLUTION. — $\phi = \dfrac{M}{\mathcal{R}} = \dfrac{12{,}560}{5} = 2512$ maxwells.

Also $B = \dfrac{\phi}{A} = \dfrac{2512}{20} = 125.6$ gausses.

It is seen that B here is the same as H above; this is correct as there is no iron in the core and $\mu = 1$, so $B = \mu H = H$.

When the magnetic circuit contains iron the reluctance is not so readily found, since the reluctivity k varies with the intensity B. It may, however, be determined satisfactorily if the quality of iron is definitely known. The following constants from Houston and Kennelly* are here given for some kinds of iron most commonly met in practice.

* *Electro-dynamic Machinery*, Art. 68, p. 65.

For ordinary *dynamo cast iron*,

$$k = 0.0026 + 0.000093\,H. \tag{66}$$

For *dynamo wrought iron*,

$$k = 0.0004 + 0.000057\,H. \tag{67}$$

For *soft iron*, according to Stoletow,

$$k = 0.0002 + 0.000056\,H. \tag{68}$$

For *cast iron*,

$$k = 0.0010 + 0.000129\,H. \tag{69}$$

For *Norway iron*,

$$k = 0.0001 + 0.000059\,H. \tag{70}$$

For *steel*, $$k = 0.00045 + 0.000051\,H. \tag{71}$$

These equations will give the reluctivity k of the kinds of iron mentioned for different intensities of field H. They apply, however, to closed magnetic circuits and are inaccurate for circuits having air gaps.

EXAMPLE.— Suppose the ring under consideration were of dynamo wrought iron, what would be its reluctance, the magnetic flux, the intensity of magnetization and the permeability?

SOLUTION. — The field as previously found is $H = 125.6$ gausses; hence

$$k = 0.0004 + 0.000057 \times 125.6$$
$$= 0.00758 \text{ oersted per cc.}$$

Hence $$\mathcal{R} = 0.00758 \times \frac{100}{20} = 0.0379 \text{ oersted.}$$

Also $$\phi = \frac{M}{\mathcal{R}} = \frac{12{,}560}{0.0379} = 331{,}400 \text{ maxwells.}$$

Therefore $B = \frac{\phi}{A} = \frac{331,400}{20} = 16,570$ gausses.

$$\mu = \frac{B}{H} = \frac{16,570}{125.6} = 132\ ;$$

or $$\mu = \frac{1}{k} = \frac{1}{0.00758} = 132, \text{ as before.}$$

For accuracy, so far as this is possible in the calculations of magnetic circuits where they are partly iron and partly air, as in horse-shoe magnets and in dynamos of all kinds, we shall have to find k for the metal portion in terms of the induction B in the metal instead of in terms of H. The reason is that though we may have the same intensity of field in the case of an open circuit as in the case of the closed ring, yet the *total* reluctance being greater, the induction in the iron is less; hence we should obtain a value too large for k by using the above formula for open magnetic circuits. Hence a formula which will give k directly in terms of B itself will be more nearly correct.

In the formula for k, call the first constant a, the second b; the general equation is then

$$k = a + bH. \tag{72}$$

We also have, from analogy to the electric circuit,

$$B = \frac{H}{k} = \mu H.$$

Whence $$H = Bk;$$

substituting above, $$k = a + bBk.$$

Hence $$k = \frac{a}{1 - bB}. \tag{73}$$

Example. — Suppose the iron ring in the last example have a gap 1 centimeter wide cut through it in one place; what effect will this have on the total reluctance and the total lines of force in the ring?

Solution. — First the reluctance of the air gap is

$$\mathcal{R}_a = \frac{l}{a} = \frac{1}{20} = 0.05 \text{ oersted.}$$

The reluctance of the iron portion is also

$$\mathcal{R}_i = k\frac{l}{a} = k \times \frac{99}{20}.$$

But k is to find. To use the formula just worked out for reluctivity in terms of B, it is observed that we must know B first. Now the air gap adds 0.05 oersted to the reluctivity of the iron which before was 0.0379, but which will be less now because the induction is less. So the total reluctance is seen to be over twice the reluctance before the gap was made. Approximately it is twice. So the flux will be approximately $\frac{1}{2}$ of the former, giving an intensity of induction B about 8000 gausses. Therefore

$$k = \frac{a}{1 - bB} = \frac{0.0004}{1 - 0.000057 \times 8000}$$
$$= 0.00074 \text{ oersted per cc.}$$

Substituting in the equation for iron reluctance,

$$\mathcal{R}_i = 0.00074 \times \tfrac{99}{20} = 0.0036 \text{ oersted.}$$

The total reluctance is

$$\mathcal{R}_a + \mathcal{R}_i = 0.05 + 0.0036 = 0.0536 \text{ oersted.}$$

Hence the total flux is

$$\phi = \frac{12{,}560}{0.0536} = 234{,}000 \text{ maxwells.}$$

$$B = \frac{234{,}000}{20} = 11{,}700 \text{ gausses.}$$

This is somewhat larger than that assumed in obtaining k, but the slight difference this will make in the iron reluctivity will not materially affect the total reluctance, since the air gap is so large a proportion.

For example, take $11{,}700 = B$ and apply in the formula for k, whence

$$k = \frac{0.0004}{1 - 0.000057 \times 11{,}700} = 0.00090,$$

which will not materially alter the *iron* reluctance, and much less the total reluctance.

The problem is most often one to find the necessary M.M.F. to supply a given flux in the air gap or through the magnet core. In this event the reluctivity at the corresponding flux density may readily be found from (73) when the quality of the iron is known.

Example. — Assuming that there are 1000 turns on the ring, how many amperes will be necessary to provide the M.M.F. as required in the previous example?

Solution. — For the air gap, if it carries 234,000 maxwells,

$$M_a = \phi_a \mathcal{R}_a = 234{,}000 \times 0.05 = 11{,}700 \text{ gilberts.}$$

For the iron portion, assuming the reluctance as estimated,

$$M_i = \phi_i \mathcal{R}_i = 234{,}000 \times 0.0036 = 842 \text{ gilberts.}$$

Therefore the total magnetomotive force is

$$M = 11{,}700 + 842 = 12{,}542 \text{ gilberts.}$$

But $$M = 1.256\, nI.$$

Therefore

$$nI = \frac{M}{1.256} = \frac{12{,}542}{1.256} = 10{,}000 \text{ amp.-turns, approximately.}$$

Hence $$I = \frac{10{,}000}{1000} = 10 \text{ amperes.}$$

EXAMPLE. — It is desired to find the M.M.F. in gilberts and the number of ampere-turns to excite the ring so that 12,000 gausses may be the air-gap density, assuming no leakage. What will be the permeability of the iron under these conditions?

SOLUTION. — We have first to find the reluctance. The reluctivity of the iron is

$$k = \frac{a}{1 - bB} = \frac{0.0004}{1 - 0.000057 \times 12{,}000}$$
$$= 0.00126 \text{ oersted per cc.}$$

The total reluctance is therefore

$$\mathcal{R} = \frac{l_a}{a_a} + k\frac{l_i}{a_i} = \frac{1}{20} + 0.00126 \times \frac{99}{20} = 0.056237 \text{ oersted.}$$

Hence

$$M = \phi\mathcal{R} = 12{,}000 \times 20 \times 0.05624 = 13{,}497 \text{ gilberts.}$$

Also $$nI = \frac{M}{1.256} = \frac{13,497}{1.256} = 10,746 \text{ amp.-turns.}$$

$$\mu = \frac{1}{k} = \frac{1}{0.00126} = 794.$$

We may also obtain H, the intensity of field, from

$$H = \frac{M}{l} = \frac{13,497}{99} = 139.$$

EXAMPLE. — Assume a coefficient of leakage of 1.2. Determine the values of the quantities as in the last example.

SOLUTION. — The flux desired in the air gap, as before, is $12,000 \times 20 = 240,000$ maxwells. The necessary flux in the ring is

$$240,000 \times 1.2 = 288,000 \text{ maxwells.}$$

$$B = \frac{288,000}{20} = 14,400 \text{ gausses.}$$

$$k = \frac{0.0004}{1 - 0.000057 \times 14,400} = 0.002232 \text{ oersted per cc.}$$

Therefore

$$\mathcal{R} = 0.05 + 0.002232 \times \frac{99}{20} = 0.06105 \text{ oersted.}$$

$$M = \phi\mathcal{R} = (14,400 \times 20) \times 0.06105 = 17,580 \text{ gilberts.}$$

Also $$nI = \frac{17,580}{1.256} = 14,000 \text{ ampere-turns.}$$

This is an increase of 3254 ampere-turns over that in the last example, caused by leakage. If the ring is wound with 1000 turns the wire must be taken large enough to carry 14 amperes of current.

It may seem that we should take only 12,000 gausses for the intensity in the air gap, while 14,400 is the intensity in the iron. But it takes M.M.F. to carry the leakage lines through the air as well as the useful ones, and to consider that only 240,000 lines passed through the air will obtain an M.M.F. too small. Although we do not know the exact path of the leakage lines, our results will be the more accurate if we consider that the whole flux set up in the core passes across the air space and obtain the M.M.F. for the latter accordingly.

40. The Lifting Power of Magnets. — As already given* the formula of Maxwell for the tractive force of magnets for their armatures when there is no appreciable air gap at the contact is

$$W = \frac{B^2 A}{8\pi}, \tag{74}$$

in which W is the weight lifted in dynes, and A is the area of polar contact in square centimeters. To reduce this to grams lifted divide by 981 and the formula becomes

$$W = \frac{B^2 A}{8\pi \times 981}. \tag{75}$$

To express the tractive force in pounds divide again by 453.6; whence, reducing the denominator,

$$W = \frac{B^2 A}{11.183 \times 10^6}. \tag{76}$$

It is to be observed that horseshoe magnets, such as are used in factories, machine shops, etc., for heavy lifting, have a double area of contact. Hence A in the formulæ is the sum of both polar surfaces.

* See Equation (51) p. 111; also see p. 122.

EXAMPLE. — An oblong horseshoe magnet has a cross section of 20 square centimeters, and the keeper, or armature, has the same shape and the same cross section, and its ends fit very closely to those of the magnet proper. The magnet is excited to an intensity across the plane of contact of $B = 15{,}000$ gausses. How many dynes, grams and pounds can be supported?

SOLUTION. —

$$W = \frac{B^2 A}{8\pi} = \frac{(15{,}000)^2 \times 20 \times 2}{8 \times 3.1416} = 358{,}097{,}000 \text{ dynes.}$$

Also

$$W = \frac{B^2 A}{8\pi \times 981} = \frac{(15{,}000)^2 \times 40}{25.13 \times 981} = 365{,}000 \text{ grams.}$$

And

$$W = \frac{B^2 A}{11.183 \times 10^6} = 805 \text{ lbs.}$$

The values, of course, include the weight of the armature.

EXAMPLE. — Let the total length of the iron circuit, magnet and keeper, be 80 cms. What M.M.F. will be necessary to magnetize to 15,000 gausses, and if the exciting current is to be 5 amperes, how many turns of wire must be put on? The magnet is made of dynamo wrought iron.

SOLUTION. — $\phi = BA = 15{,}000 \times 20 = 300{,}000$ maxwells. The reluctivity is

$$k = \frac{a}{1 - bB} = \frac{0.0004}{1 - 0.000057 \times 15{,}000}$$
$$= 0.0028 \text{ oersted per cc.}$$

The reluctance is, using a instead of A,

$$\mathcal{R} = k\frac{l}{a} = 0.0028\,\frac{80}{20} = 0.0112 \text{ oersted.}$$

Hence $M = \phi \mathcal{R} = 300{,}000 \times 0.0112 = 3360$ gilberts.

Also

$$nI = \frac{3360}{1.256} = 2675 \text{ amp.-turns.}$$

$$n = \frac{2675}{5} = 535 \text{ turns of wire.}$$

EXAMPLE. — It is not economical to excite magnets much beyond 16,000 gausses. Show to how many pounds per square inch this is equivalent.

SOLUTION. — For pounds per square centimeter we have from (76)

$$W = \frac{B}{11.183 \times 10^6} = \frac{(16{,}000)^2}{11.183 \times 10^6} = 22.9 \text{ pounds.}$$

One sq. in. = 6.45 sq. cm. Hence per sq. in.,

$$W = \frac{B^2 \times 6.45}{11.183 \times 10^6} = 147.6 \text{ lbs.}$$

Thompson gives the economical limit of 150 lbs. per square inch. This will be equivalent to an induction of

$$B = \sqrt{\frac{150 \times 11.183 \times 10^6}{6.45}} = 16{,}120 \text{ gausses.}$$

For traction purposes we may, in general, figure on 150 lbs. to the square inch, or about 24 lbs. per square centimeter. For cast iron B should not be over 6,000 or 7,000 gausses, and W about 28 lbs. per square inch. These values given, we may find at once the required cross section for any given weight to be supported.

EXAMPLE. — Let it be required to design a horseshoe magnet to carry 2 tons.

SOLUTION. — 2 tons = 2 × 2240 = 4480 lbs.

$A = 4480 \div 150 = 30$ sq. in.; or 15 sq. in., polar area.

Since the magnetic circuit should be as short as possible consistent with sufficient winding space, we shall take the length of magnet to be 20 inches, a straight armature 14 inches long, and assume the material to be Norway iron. Whence

$$k = \frac{a}{1 - bB} = \frac{0.0001}{1 - 0.000059 \times 16{,}000}$$

$$= 0.0018 \text{ oersted per cc.}$$

$$\mathcal{R} = k\frac{l}{a} = 0.0018 \times \frac{34 \times 2.54}{15 \times 6.45} = 0.0016 \text{ oersted.}$$

Hence

$$M = \phi\mathcal{R} = 0.0016 \times 16{,}000 \times 96.7 = 2475 \text{ gilberts.}$$

$$nI = \frac{M}{1.256} = \frac{2475}{1.256} = 1980 \text{ amp.-turns.}$$

EXAMPLE. — Suppose storage batteries capable of giving 5 amperes are used to excite the above magnet. Determine the length, size and resistance of the wire to be used. Also approximate the space to be allotted to the winding.

SOLUTION. — Thompson gives ½ inch as the maximum depth of winding for ordinary magnets, and if this be reached only about 1 inch of winding space need be used for each 20 inches of the iron circuit. In this case there are required

$$1980 \div 5 = 400 \text{ turns, approximately.}$$

The wire must be large enough for 5 amperes without overheating. From the table of safe carrying capacities we find No. 16 wire is sufficient. This has 50.8 mils diameter bare, or about 67 mils double cotton covered. Take winding space 2 inches = 2000 mils. In one layer there will be

$$2000 \div 67 = 30 \text{ turns.}$$

Hence there must be in depth

$$400 \div 30 = 13 \text{ layers.}$$

This gives a depth of

$$13 \times 67 = 871 \text{ mils} = 0.87 \text{ inch,}$$

which is above Thompson's limit. Hence to reduce depth we must allow more space, say 3 inches = 3000 mils. In one layer there will be

$$3000 \div 67 = 45 \text{ turns.}$$

And

$$400 \div 45 = 9 \text{ layers deep.}$$

The depth of winding now is

$$9 \times 67 = 603 \text{ mils} = 0.6 \text{ inch.}$$

The average length of one turn is

$$(\sqrt{15 \div 0.7854} + 0.6) \times 3.1416 = 15.7 \text{ inches.}$$

Total length will be

$$\frac{15.7 \times 400}{12} = 523 \text{ feet.}$$

The resistance will be

$$R = k\frac{l}{d^2} = 10.79 \times \frac{523}{(50.8)^2} = 2.18 \text{ ohms.}$$

It has been found that however closely the armature fits against the poles the reluctance of the contact is still

equivalent to 0.003 to 0.004 centimeter of air. Take 0.0035 centimeter as a fair average.

EXAMPLE. — Determine the M.M.F. for the magnet considered above, taking account of the contact reluctance.

SOLUTION. — The reluctance of the equivalent air space is

$$\mathcal{R} = k\frac{l}{a} = \frac{0.0035}{96.7} = 0.000036 \text{ oersted.}$$

Therefore the total reluctance is

$$\mathcal{R} = 0.0016 + 0.000036 \times 2 = 0.0017 \text{ oersted.}$$

$$M = \phi\mathcal{R} = 16{,}000 \times 96.7 \times 0.0017 \doteq 2630 \text{ gilberts.}$$

$$nI = 2630 \div 1.256 = 2100.$$

This makes an addition of

$$2100 - 1980 = 120 \text{ ampere-turns,}$$

due to the contact reluctance, or $\frac{120}{5} = 24$ turns of wire additional. We might neglect this slight increase in the reluctance in the calculation by slightly increasing the exciting current. In this example the current would need to be increased $\frac{1}{3}$ ampere.

41. **Temperature and Magnetism.** — If a wire nail be magnetized, then heated to redness, tests will show that it has thus lost all trace of its former magnetism. Experiment is also able to determine the rate at which the intensity of magnetism decreases with the rise of temperature. Let a be the decrease in the magnetic moment of a magnet (which is proportional to the magnetic strength) per degree of temperature per unit of magnetic moment; t the rise of temperature, M_0 the magnetic moment at 0° C.

The decrease is then $M_o \times at$. Hence the magnetic moment M_t at any increase of temperature t will be

$$M_t = M_o - M_o \times at = M_o(1 - at). \qquad (77)$$

This formula is similar to that for the resistance of a wire at any increase of temperature above that at which the resistance is given, except that the temperature coefficient of resistance is usually positive, while the *temperature coefficient* of *magnetism* a as given in the formula (77) is negative. This coefficient is small, so that a considerable change of temperature is necessary to appreciably affect the strength of the magnet.

Example. — The magnetometer needle stands in the magnetic meridian; a small 4-inch magnet is placed in an oil bath in an east-west position at a short distance west from the center of the magnetometer needle. The temperature of the oil bath is 20° C. and the deflection of the magnetometer 150 millimeters after the magnet has acquired the temperature of the oil. The oil is now heated slowly to 50° C. and the deflection becomes 148. Find the *temperature coefficient* of the magnet.

Solution. — The magnetic moment $M = ml$; *i.e.* the pole strength times the length, and hence the pole strengths m are proportional to the deflections. Hence from (77)

$$148 = 150\,[1 - a(50 - 20)] = 150 - 4500\,a.$$

Therefore

$$a = \frac{148 - 150}{-4500} = 0.00044.$$

42. Hysteresis and Eddy Currents. — EXAMPLE. — Compare the hysteresis losses in two transformers having the same induction but in one of which the frequency is 60 and in the other it is 120.

SOLUTION. —
$$\frac{W_h'}{W_h''} = \frac{h'f'B'^{1.6}}{h''f''B''^{1.6}} = \frac{f'}{f''} = \frac{60}{120} = \frac{1}{2},$$
since $h' = h''$ and $B' = B''$. That is, since the loss varies according to the frequency, the first will have one-half the loss of the second.

EXAMPLE. — Find the watts lost due to hysteresis in a transformer of 4000 cubic centimeters in which $B = 3000$ gausses, $f = 100$, and the hysteretic constant $h = 20 \times 10^{-11}$ watts per cubic centimeter per cycle.

SOLUTION. — The watts lost per cc. are given in the formula
$$W_h = hfB^{1.6},$$
and for V cu. cm.,
$$VW_h = VhfB^{1.6} = 4000 \times 20 \times 10^{-11} \times 100 \times \overline{3000}^{1.6}$$
$$= 8 \times 10^{-5} \times \overline{3000}^{1.6}.$$

We find now from a table of logarithms the logarithm of 3000, multiply it by 1.6 and find the number corresponding; this gives the 1.6th power of 3000. Proceeding,
$$3000 = 100 \times 30,$$
and $\log 3000 = \log 100 + \log 30 = 2 + 1.47712 = 3.47712.$
$$3.47712 \times 1.6 = 5.56339$$
which is found to be the log of 365924; the latter is

therefore the 1.6th power of 3000; or $\overline{3000}^{1.6} = 365924$. Substituting above, we have

$$VW_h = 8 \times 10^{-5} \times 3.65924 \times 10^5 = 29.27 \text{ watts.}$$

EXAMPLE. — Find the watts lost in the transformer considered due to eddy currents if the plates, or laminæ, are 12 mils thick.

SOLUTION. — The formula for eddy currents is

$$W_e = V(tfB)^2 \times 10^{-16}.$$

Therefore

$$W_e = 4000(12 \times 100 \times 3000)^2 \times 10^{-16} = 5.18 \text{ watts.}$$

EXAMPLE. — If a certain test of this transformer shows 6 watts lost due to eddy currents, what is the induction?

SOLUTION. — From the above formula,

$$B = \sqrt{\frac{W_e}{V \times (12 \times 100)^2 \times 10^{-16}}} = \sqrt{\frac{6}{4000 \times (1200)^2 \times 10^{-16}}}$$

$= 3227$ gausses.

43. **Original Problems.** — 1. If a magnetic needle make 24 oscillations per minute at a point where the horizontal intensity of the earth's field is 0.2 dyne, what is the horizontal intensity at a place where the same needle makes 22 oscillations per minute? $H = 0.168$ dyne.

2. A given magnetic needle makes 24 vibrations in the earth's field alone. When magnet *A* is placed in the meridian at 20 centimeters from the needle, the latter makes 31 vibrations per minute. When magnet *B* is placed in the same position and at the same distance the needle makes 28 vibrations per minute. Compare the forces due to *A* and *B*. $A = 1.85\ B$.

3. Compare the pole strength of A, 30 swings per minute, with that of B, 36 swings per minute, when the needle makes 24 swings per minute in the earth's field.

$$\frac{A}{B} = \frac{9}{20}.$$

4. What is the magnetic density when a spherical surface whose radius is 5 centimeters has at its center a magnetic pole whose strength $m = 200$? How many maxwells of flux pass through this surface?

$B = 8$ gausses.
$\phi = 2512$ maxwells.

5. A ring of annealed iron is required to have 600,000 maxwells of flux at a density of 15,000 gausses. What must be its cross section, and what is its permeability?

$A = 40$ sq. cm.
$\mu = 500$.

6. What is the field intensity in the last problem, and how many amperes will be necessary to produce the field if the coil has 50 turns per centimeter?

$H = 30$ gausses.
$I = 0.48$ ampere.

7. The cross section of the rod used in a permeameter for testing permeabilities is 1 sq. cm., and requires a force of 10,000 grams to pull it off when the coil having 25 turns per centimeter carries 10 amperes. What is the permeability of the test piece? $\mu = 50$.

8. If the rod is required to have a permeability of 30, how many grams of force will pull it off when the current is 15 amperes? $W = 8000$ grams.

9. Determine the reluctance of a brass ring whose circumference is 125 centimeters, and cross section ·2 sq. cm., if an air gap of 2 cms. length be cut across it.

$\mathcal{R}$ = 62.5 oersteds.

10. What M.M.F. will be required to produce a density of 200 gausses in the ring, and how many ampere-turns must be wound on the ring for this purpose?

M = 25,000 gilberts.
nI = 20,000 amp.-turns.

11. What will be the field intensity if the ring is wound for 25 amperes of current? H = 200 gausses.

12. Suppose the above ring were of wrought iron and have no air gap. Find its reluctivity, reluctance, flux density and permeability. k = 0.0118 oersted per cc.

$\mathcal{R}$ = 0.7375 oersted.
B = 16,800 gausses.
μ = 84.

*13. How many turns of wire for 2 amperes must be wound uniformly on this ring to produce a flux of 20,000 maxwells? What will be the field intensity under these conditions? n = 465 turns.

H = 9.3 gausses.

14. Suppose 1 cm. of the ring be cut out, thus forming an air gap. How many turns of wire for 5 amperes exciting current must be wound on to give an intensity of 10,000 gausses in the iron and 9090 gausses in the air gap, thus making the leakage coefficient 1.1?

n = 1784 turns.

15. Find the permeability of the iron in the last problem. μ = 1075.

* See note on p. 147.

*16. A magnet of square horseshoe shape has the following dimensions: total length of iron 60 cms.; cross section 25 sq. cms.; length of air gap 0.075 cms.; area of air gap 25 sq. cms. The armature is drum, or cylindrical, in shape as in motors and dynamos, and is 7 cms. in thickness from air gap to air gap. The magnet is wound with wire for 2.5 amperes to furnish a flux of 350,000 lines available through the armature and air gaps, coefficient of leakage estimated at 1.2. How many turns of wire will it take and what winding space will be necessary, allowing 1000 circular mils per ampere and 25% for insulation of wires, and permitting a depth of wire of 1.5 inches? Also how many feet of wire will be required?

For magnet M.M.F. = 5000 gilberts.
For armature M.M.F. = 35 gilberts.
For air gap M.M.F. = 2520 gilberts.
Total M.M.F. = 7555 gilberts.
n = 2418 turns of wire.

Wire is 2500 cir. mils = No. 16 = 50.8 mils bare = 63 mils covered.

Wire space = 6.3 inches.
Length of wire = 2829 ft.

17. How many dynes, grams and pounds may be supported on the hook of a lifting magnet having the following constants: induction 16,000 gausses across polar contact; weight of armature 20 lbs.; area of pole faces 25 sq. cms. each?

$$W = 5.12 \times 10^8 \text{ dynes.}$$
$$= 521.7 \text{ kilos.}$$
$$= 1150 \text{ pounds.}$$

From these subtract the weight of the armature.

* Work by curves, p. 229, or tables, p. 298.

18. Assuming that 16,000 gausses is a fair limit to economical induction, what must be the area of a magnet to support 1 ton (2000 lbs.) including the weight of the keeper? $A = 43.68$ sq. cms.
$= 6.77$ sq. ins.

19. If a test shows that a lifting magnet is carrying 1600 lbs., while its area of contact is 10 sq. ins., what must be the induction across the area of contact?

$B = 11{,}800$ gausses.

*20. If the total length of the iron circuit in problem 17 is 12 inches and the material is dynamo wrought iron, how many ampere-turns will be necessary to give the proper excitation? By curve, $nI = 1391$ amp.-turns.

21. If a 6-ampere exciting circuit is available, how many turns and what size of wire will be necessary for the magnet in problem 18, if we may assume the total length of iron to be 20 inches?

By curve, $n = 380$ turns of No. 12.

22. How much space must be provided for the wire above, if we may allow a depth of 1 inch? Also what will be the length of wire? Wire space = 3.8 inches.
Length = 386 feet.

23. Required to design a horseshoe lifting magnet of circular cross section to carry 3 long tons. Specify all values for the construction of the magnet of Norway iron, taking a current of 5 amperes.

* The answers in this and the next problem cover allowance for air gap.

Area = 22.4 sq. in., cross section.
Length = 32 inches = 80 cms., estimated.
Diam. = 5.34 inches = 13.5 cms.
Depth of wire = 0.5 inch.
Size of wire = No. 14 = 80 mils d.c.c.
Length of wire = 735 feet.
Winding space = $6\frac{1}{2}$ inches total.
Ampere-turns = 2400; n = 480 turns.

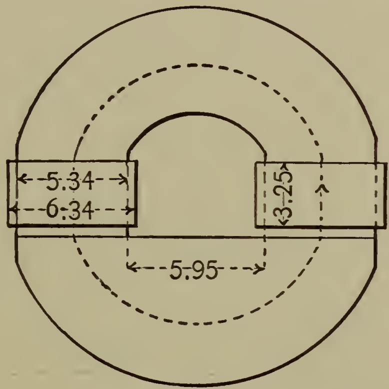


Fig. 9.

24. Find the length and the resistance of the wire in the last problem if allowance be made for the reluctance of the contact area. Also determine the depth of wire if the same space be allowed as before.

$nI = 2490$; $n = 498$.
$L = 763$ ft.; $R = 2.2$ ohms.
Wire is No. 14 B. & S.

25. Give the design for a lifting magnet to be used in a factory where weights up to 1200 pounds are to be handled, constructing it of dynamo iron, allowing for joints, and having 3 amperes for excitation. Use tables.

$L = 40$ cm., total; $L = 28$ cm., horseshoe.
$A = 25.8$ sq. cm. $= 4$ sq. in.
$nI = 2012$; $n = 671$; wire = No. 16 d.c.c.
Depth = 8 layers = 0.5 in.; space = 5.4 in., or 2.7 in. each side.
L of wire = 526 ft.; $R = 2.7$ ohms; wt. iron = 8 kilos.

26. A steel magnet immersed in an oil bath at 15° C. at a certain distance from the magnetometer causes a deflection of 200 mm. Assuming its temperature coefficient to be 0.00045, to what temperature must it be heated theoretically to lose $\frac{1}{4}$ its magnetism?

$t = 570.5°$ C.

27. Find the temperature coefficient of a magnet if when placed at a certain distance from the magnetometer, its axis on an east and west line, it gives a deflection of 200 mm. at 20° C., but when heated to 100° C. it produces a deflection of 192 mm.

$a = 0.0005$.

28. How much greater is the loss in a transformer for a frequency of 120 with an induction of 4000 gausses made of iron such that the hysteretic constant is 21×10^{-11}, than in one of the same capacity and with the same induction, for a frequency of 100, and having a hysteretic constant of 20×10^{-11}?

Hysteresis loss first $= \frac{63}{50}$ of second.

29. Determine the total hysteresis loss of a transformer in watts, its volume being 6000 cc., induction 3200 gausses, frequency 120, hysteretic constant 20×10^{-11} watts per cc. per cycle.

$$VW_h = 58.42 \text{ watts.}$$

SUGGESTION. — $\log 3200 = \log 100 + \log 32$
$= 2 + 1.50515.$
$\log x = 1.6 \times 3.50515 = 5.60824.$

Use in $hfB^{1.6}$, $B^{1.6} = x = \overline{3200}^{1.6} = 405730.$

30. Calculate the eddy current loss in the above transformer, the laminæ being 10 mils thick.

$$W_e = 8.85 \text{ watts.}$$

31. With a certain exciting current this transformer experienced an eddy current loss of 8 watts. What was the intensity of induction? $B = 3043$ gausses.

32. Find the total core loss in a transformer of 5000 cubic centimeters, frequency 100, induction 3000 gausses, hysteretic constant of the iron 20×10^{-11}, thickness of plates 13 mils.

$W_h = 36.59$ watts.
$W_e = 7.6$ watts.
Total $= 44.2$ watts.

NOTE. Instead of the more tedious reluctance method of calculating the required ampere-turns for a magnetic circuit, as described in this Chapter, and originally used in the solution of the problems in this section, it will be more convenient to use the curves given on page 229. Obtain B from the given conditions, find H corresponding from the proper curve, whence $ni = \frac{Hl}{1.256} = .8\, Hl$, approximately, where l is length in centimeters of the particular portion being calculated. The total ampere-turns will then be the sum of the several partial ampere-turns. See also tables on page 298 for $\frac{nI}{l}$.

IX.

THE E.M.F. OF DYNAMOS AND MOTORS.

44. **Bipolar, Direct Current Machines.** — There are three elements on which the electromotive force of a dynamo machine depends; namely, the magnetic flux through the armature, the number of armature conductors and the speed of rotation of the armature. All this may be summed up by saying *the electromotive force depends on the rate of change of the magnetic flux through the armature loops.*

Representing the flux in maxwells by ϕ, the number of armature conductors counted all the way around by n, the speed in revolutions per second by v, the E.M.F. set up in an armature will be, in C.G.S. units,

$$E = \phi n v. \tag{78}$$

Or expressing E in volts the E.M.F. is

$$E = \frac{\phi n v}{10^8}. \tag{79}$$

Evidently any one of the four quantities in this formula may be found when the other three are given, or when conditions are given making it possible to determine the other three. Thus

$$\phi = \frac{10^8 E}{n v}; \quad n = \frac{10^8 E}{\phi v}; \quad v = \frac{10^8 E}{\phi v}.$$

EXAMPLE. — A dynamo, drum armature, has 50 commutator segments and 50 coils of wire No. 10, two turns to each coil, which are rectangular in shape, $8'' \times 12''$; the density of flux through the armature which runs at 2500 r.p.m. is 10,000 gausses. What E.M.F. does it generate?

SOLUTION. — There are $50 \times 2 = 100$ turns, and since for each coil there are two surface wires, there are in this case $100 \times 2 = 200$ wires, or surface conductors. $v = 2500 \div 60 = 41\frac{2}{3}$ r.p.s.; $\phi = 10{,}000 \times 8 \times 12 \times \frac{25}{4} =$ 6,000,000 maxwells approximately. Hence

$$E = \frac{\phi n v}{10^8} = \frac{6 \times 10^6 \times 200 \times 41\frac{2}{3}}{10^8} = 500 \text{ volts.}$$

EXAMPLE. — How many coils of wire of 2 turns each would it require on this armature to generate 250 volts?

SOLUTION. —

$$n = \frac{10^8 \times E}{\phi v} = \frac{10^8 \times 250}{6 \times 10 \times 41\frac{2}{3}} = 100 \text{ conductors.}$$

Hence there will be $\frac{100}{2} = 50$ coils of one turn each or 25 coils of 2 turns each. It would perhaps be better to use 50 coils of one turn each and 50 commutator bars.

EXAMPLE. — If we wish to change the dynamo pulley so as to make a 200 volt machine of it, the pulley must be selected so as to give what speed in r.p.m.?

SOLUTION. —

$$v = \frac{10^8 \times E}{\phi n} = \frac{10^8 \times 200}{6 \times 10 \times 100} = 33\tfrac{1}{3} \text{ r.p.s.}$$

Hence the speed of the pulley should be $33\frac{1}{3} \times 60 =$ 2000 r.p.m.

45. **Alternators.** — Alternators usually have all the armature coils connected in series instead of the two halves in parallel as in bi-polar direct current machines. The E.M.F. will therefore be double, other conditions being the same. Hence a two-pole alternator would give an E.M.F.

$$E = \frac{2\,\phi n v}{10^8}. \tag{80}$$

If there are p pairs of poles, the rate of change of magnetic lines will be p times as great, and

$$E = \frac{2\,\phi n v p}{10^8}. \tag{81}$$

As previously stated the *average* electric pressure is $\frac{2}{\pi} = 0.636$, while the *effective* pressure is $\frac{1}{\sqrt{2}} = 0.707$. Hence the formula (81) must be increased by the ratio of these, or by $\frac{1}{\sqrt{2}} \div \frac{2}{\pi} = 1.11$. We shall call this value k, whence

$$E = \frac{2\,\phi n v p k}{10^8}. \tag{82}$$

In practical machines the value of k will vary somewhat from the theoretical value, depending on the width of poles, pitch and nature of windings. It generally ranges from 1 to 1.2. But there will be on the other hand a certain amount of opposing E.M.F. in the opposite sides of each coil because of their width, the amount depending on the ratio of the width and pitch of poles to the width of the coils. This tends to reduce k, so that as an average it will not vary much from 1.11.

In alternators multiple wound the electromotive force is reduced to that produced only by the number of coils in series. If half are in series, then

$$E = \frac{\phi n v p k}{10^8}. \tag{83}$$

It is to be observed that n is always the number of conductors counted all the way round the surface of the armature; and ϕ is the maxwells of flux out of one pole into the armature.

We may modify (82) by substituting f, the frequency, for the product vp to which it is equal; and instead of k put its value, 1.11. We then have

$$E = \frac{2.22\, \phi n f}{10^8}. \tag{84}$$

If it is desired to find the flux which each pole must carry in order to produce a certain E.M.F. at a given frequency when the number of armature conductors is fixed, we have from (84)

$$\phi = \frac{E \times 10^8}{2.22\, n f}. \tag{85}$$

For the number of armature conductors,

$$n = \frac{E \times 10^8}{2.22\, \phi f}. \tag{86}$$

For the frequency from which v may then be obtained,

$$f = \frac{E \times 10^8}{2.22\, \phi n}. \tag{87}$$

Example. — Suppose the conductors of the two-pole machine represented in the first example under section

50 were all in series and the opposite ends connected to two rings so as to supply an alternating current. What would be the E.M.F.?

SOLUTION. —

$$E = \frac{2\,\phi n v}{10^8} = \frac{2 \times (6 \times 10^6) \times 200 \times 41\frac{2}{3}}{10^8} = 1000 \text{ volts.}$$

EXAMPLE. — Determine the electromotive force of a 6-pole alternator running at 2100 r.p.m., the flux being 24×10^5 maxwells and there being 200 wires on the armature.

SOLUTION. —

$$E = \frac{2\,\phi n v p k}{10^8} = \frac{2 \times 24 \times 10^5 \times 200 \times 35 \times 3 \times 1.11}{10^8}$$

$$= 1118.8 \text{ volts.}$$

EXAMPLE. — What must be the number of maxwells of flux furnished by each pole of a machine similar to the last for 560 volts, and what cross section of poles? Also what width of poles to be equal to one-half the pitch?

SOLUTION. —

$$\phi = \frac{E \times 10^8}{2.22\, nf} = \frac{560 \times 10^8}{2.22 \times 200 \times 3 \times 35} = 1{,}201{,}200 \text{ maxwells.}$$

For dynamo poles B may be, say 12,000 gausses. Hence

$$A = \frac{\phi}{B} = \frac{1{,}201{,}200}{12{,}000} = 100 \text{ sq. cm.}$$

The circumference of the pole circle is very nearly 80 cms. Since there are 6 poles and the distance apart is equal to the width, each must be $80 \div 12 = 6.66$ cms.

The length of pole face and armature core must then be $100 \div 6.66 = 15$ centimeters.

46. Multipolar Direct Current Dynamos. — The usual method of winding Gramme ring armatures is to wind the different coils the same way continuously around the ring. Now when the coils are joined together, the end of the first may be joined to the beginning of the second, the end of the second to the beginning of the third, etc. There will then be as many brushes and as many armature circuits as poles. This is called the *multiple winding*. Thus a 4-pole machine requires 4 brushes — two pairs at right angles, and 4 circuits in the armature all in parallel. Evidently the brushes of like polarity may be connected together, thus putting all the current in a single external circuit. Also the coils may be so cross connected at the commutator end of the armature as to require but two brushes set at an angular distance apart equal to the pole pitch. The E.M.F. of such an armature is

$$E = \frac{\phi n v}{10^8}. \qquad (88)$$

As before, ϕ is the maxwells of armature flux from each pole, n the number of armature conductors counted all the way around, and v the speed in r.p.s.

On the other hand the coils may be joined by connecting the end of the first to the end of the second, the beginning of the second to the beginning of the third, the end of the third to the end of the fourth, the beginning of the fourth to the beginning of the next, etc. In a 4-pole and 4-coil machine, the beginning of the fourth

is connected to the beginning of the first, thus completing the closed circuit. This is called the *series winding.* It requires two brushes and has two circuits through the armature in parallel. If there are p pairs of poles the E.M.F. for the series winding is

$$E = \frac{\phi n v p}{10^8}. \qquad (89)$$

In both cases there are as many parts on the commutator as coils, and a connection to the commutator is made at each junction of two coils.

For drum armatures the *multiple winding* is made by putting each coil on a chord so as to cover an angular space on the armature about equal to the pitch of the poles. The coils overlap each other, and the winding is done exactly as for 2-pole machines.

The *series winding* is made by winding forward along the drum, back at an angular distance from first wire about equal to the pitch, forward at the same angle from last, and so on till the surface of the armature has been turned once over; if several turns are to be put in each coil they must all be wound successively in the same slots. When one coil is done, the next is put on in its proper slots in the same way as the first, etc., until all are on. Then the end of the first is joined to the beginning of the next, etc. This arrangement requires but two brushes. The winding, if spread out, is a wave, like this:

Hence the name, *wave winding.*

The same formulæ (88) and (89) apply to the multiple and series, respectively.

When the number of pairs of poles is even, the two brushes in the series winding will be at an angular distance apart equal to the pole pitch. If the number of pairs be odd, however, the brushes will be diametrically opposite.

Example. — Two drum armatures for 6-pole machines are exactly alike, except one is *multiple* wound, while the other is *series*. The following constants are known: Flux 6×10^6 maxwells; surface conductors 500; speed 20 r.p.s. Determine their respective E.M.F.'s.

Solution. — For the multiple wound,

$$E = \frac{\phi n v}{10^8} = \frac{6 \times 10^6 \times 500 \times 20}{10^8} = 600 \text{ volts.}$$

For the series wound,

$$E = \frac{\phi n v p}{10} = \frac{6 \times 10^6 \times 500 \times 20 \times 3}{10^8} = 1800 \text{ volts.}$$

Example. — What will be their respective resistances and current capacities?

Solution. — If wound with wire of the same size the multiple wound can carry 3 times as much current as the series wound, because with 6 brushes and 6 circuits in parallel, each circuit carries $\frac{1}{6}$ of the total output in amperes. On the other hand, the series wound armature has 2 circuits in parallel, each carrying $\frac{1}{2}$ of the total current in amperes. Hence the former can carry 3 times

as much as the latter. The output in watts is the same in both cases.

The resistance of the multiple wound will be, for the reasons just stated, $\frac{1}{9}$ of that of the series winding. The latter has $\frac{1}{4}$ the resistance of all its armature wire, since two halves of it are in parallel. The former has $\frac{1}{36}$ the resistance of all its wire, since its 6 sixths are in parallel.

EXAMPLE. — A machine is required to give 560 volts and to run at a speed of 1800 r.p.m. Suppose it is built with 6 poles, what area must each pole face present to the air gap, not allowing for leakage?

SOLUTION. — From (89)

$$\phi = \frac{E \times 10^8}{nvp}.$$

Assuming 400 armature conductors *series* wound,

$$\phi = \frac{560 \times 10^8}{400 \times 30 \times 3} = 15.55 \times 10^6 \text{ maxwells.}$$

Allow $$B = 15{,}550 \text{ gausses,}$$

then $$A = \frac{15.55 \times 10^6}{15.55 \times 10^3} = 1000 \text{ sq. cms.}$$

EXAMPLE. — What will be the diameter of the armature if the pole face may be taken $2\frac{1}{2}$ times as long as wide, the pole width being equal to one-half the pitch?

SOLUTION. — The width of the pole face is

$$\sqrt{1000 \div 2.5} = 20 \text{ cm.}$$

The length of the face is

$$1000 \div 20 = 50 \text{ cm.}$$

This is also the length of the armature core. The circumference of the pole circle is therefore $20 \times 2 \times 6 = 240$ cm. The diameter is $240 \div 3.1416 = 76.4$ cm. Allowing an air gap of 0.2 cm., the diameter of the armature must be 76 centimeters.

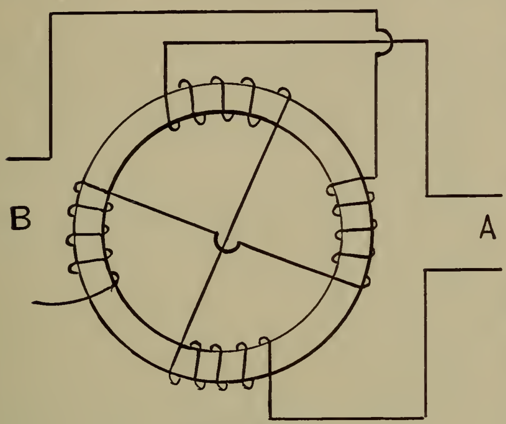


Fig. 10 (a).

47. Polyphase Armatures.—Armatures wound *two-phase* are usually connected in one of two ways. First, the two windings which are put on the core ninety electrical degrees apart are connected independently each to its pair of rings as shown in Fig. 10.

This requires four wires for its external circuits. The armature winding may be slightly modified by joining the circuits at their middle points; that is, at the points where they cross over in the figure.

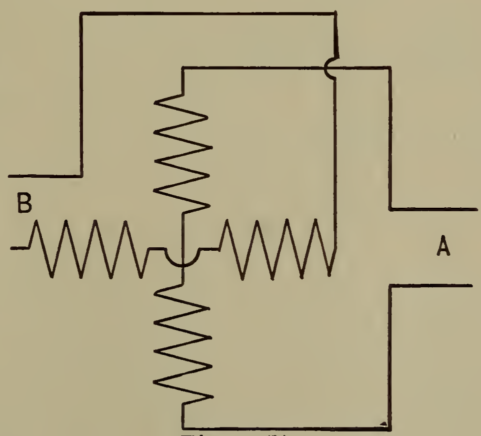


Fig. 10 (b).

Second, the finishing ends of each set of windings may be joined together, thus requiring but three wires for the external

circuit. See Fig. 11. In the first case, that of independent external circuits, whether the coils are con-

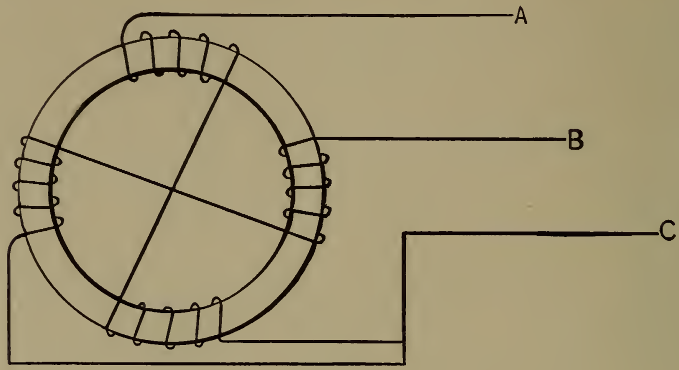


Fig. 11 (a).

nected at the center or not, the voltage of each of the circuits *A* and B will be

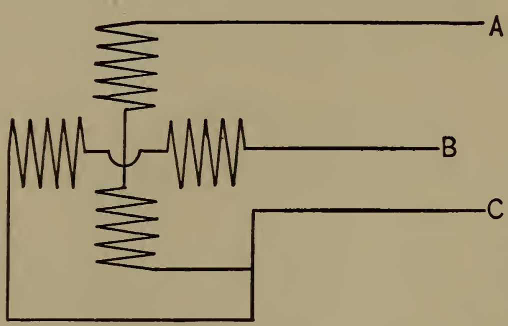


Fig. 11 (b).

$$E = \frac{2\,\phi \frac{1}{2} n v p k}{10^8}. \quad (90)$$

or

$$E = \frac{1.11\,\phi n f}{10^8}. \quad (91)$$

In these formulæ n is the number of conductors on the armature surface in *both* sets of coils, $f = vp$ is the frequency, and ϕ is the flux from each pole. It is obvious that each armature circuit constitutes an independent alternator, and hence the same formula as for plain alternators applies, (82) and (84), if we count n the number of surface conductors in each phase.

In the second case, Fig. 11, the E.M.F. between either

A or B and the common line C, that is, the machine E.M.F. will be

$$E = \frac{2.22\,\phi \frac{n}{2} vp}{10^8} = \frac{1.11\,\phi nf}{10^8}.$$

This is the same as the first case. But between A and B the E.M.F. is the resultant of two E.M.F.'s at right angles, or 90 degrees apart; hence

$$E' = \sqrt{2}\,E,$$

which substituted in (90) gives

$$E = \frac{\sqrt{2} \times 2.22\,\phi \frac{n}{2} vp}{10^8} = \frac{1.57\,\phi nf}{10^8}. \qquad (92)$$

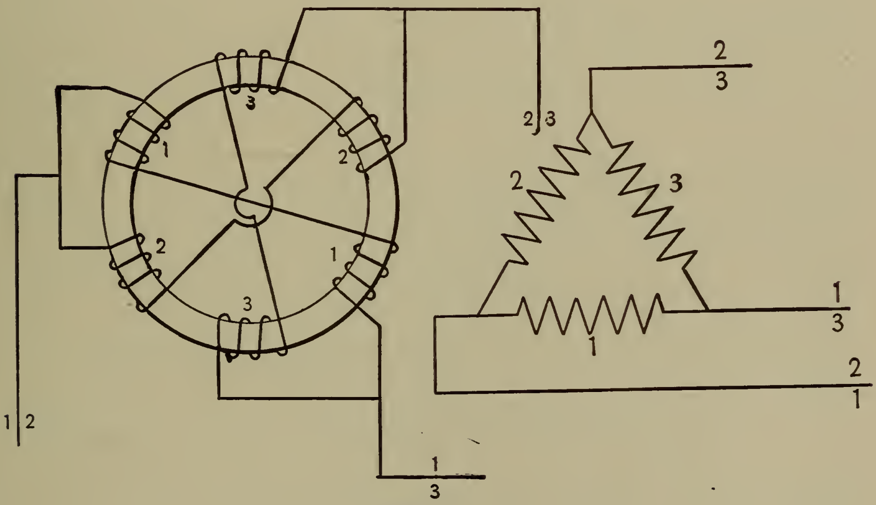


Fig. 12.

If the current in each circuit as measured by an alternating current ammeter placed in the wire A or B be I, then the current in the common wire C is the geometric sum, or resultant of the two equal currents at right angles; namely $I' = \sqrt{2} \times I$. This assumes the circuits to be properly balanced.

Armatures wound *three-phase* are also usually connected in one of two ways. First, the coils for each of the three phases are put on 120 electrical degrees apart, and connected as shown in Fig. 12, 1 to 2, 2 to 3, 1 to 3.

This is the *mesh* arrangement, requires 3 rings, 3 wires for the external circuit, and gives an E.M.F. between any two wires which is equal to the E.M.F. set up in each phase. This is

$$E = \frac{2.22\,\phi \frac{n}{3} f}{10^8}. \tag{93}$$

This may be written

$$E = \frac{0.74\,\phi n f}{10^8}.$$

n is the number of surface conductors counted in all the phases, and $f = vp$, as before. The current in any line wire is $\sqrt{3}$ times the current in any one-phase winding in the armature. If I' be the current in coil 1, the current measured by an amperemeter in any line will be

$$I = \sqrt{3}\,I' = 1.732\,I'. \tag{94}$$

This relation comes from the fact that the line current is the resultant of two equal armature currents 120 degrees apart; that is, it is the third side of an isosceles triangle opposite the angle of 120°. Hence it is $\sqrt{3}$ times each of the two equal sides.

Second, one end of each winding is connected to a common junction, the other ends being joined respec-

tively to one of the three rings according to the figure below, Fig. 13.

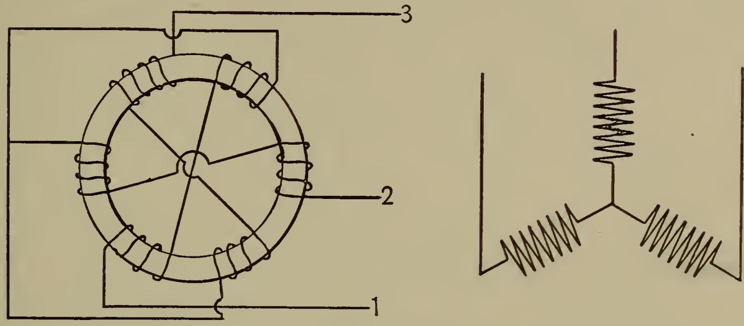


Fig. 13.

This is the *star* arrangement, and the E.M.F. of the machine, that is, the E.M.F. between any two rings, is the geometric sum of the two E.M.F.'s at 120° apart in phase. Hence

$$E = \frac{\sqrt{3} \times 2.22 \times \phi \times \frac{1}{3} nf}{10^8}. \tag{95}$$

Reducing,

$$E = \frac{1.28\, \phi nf}{10^8}. \tag{96}$$

n is the whole number of surface conductors in all the phases. If E' be the E.M.F. in any one of the three armature windings, then the machine E.M.F. is

$$E = \sqrt{3} \times E' = 1.732\, E'. \tag{97}$$

The current in any wire of the circuit is the same as the current in the armature coils of the corresponding phase.

Armatures of the *monocyclic* type which are virtually polyphase when used for running motors are wound just as any ordinary single-phase alternator, that is, with a continuous winding, the ends being connected each to a

ring; but in addition another *teaser* winding is put on as follows: one end is connected to the middle of the main winding, and its coils, each having *one-fourth* as many turns as those of the main winding, are placed 90 electrical degrees from them. The other end is then connected to a third ring placed between the other two. For lighting only the main winding and the outside rings are used. For power the three rings and three circuit wires are used. The windings are related as in the diagram, Fig. 14.

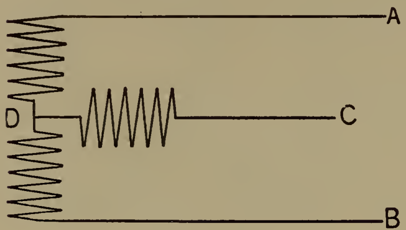


Fig. 14.

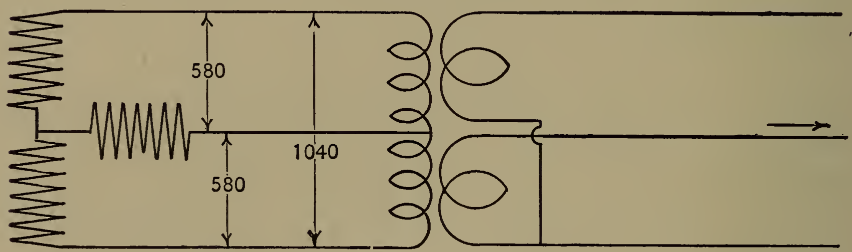


Fig. 15.

Between A and B is the main machine voltage supplied to transformers for the lighting circuits. When induction motors are to be supplied, a third wire from ring C and two transformers are required. Fig. 15 represents the method of connecting up the transformers for power purposes on monocyclic circuits.

The E.M.F. for lighting, that is, between A and B, is

$$E = \frac{2\,\phi n v p k}{10^8}. \tag{98}$$

The E.M.F. between A and C or B and C is the resultant of two E.M.F.'s at right angles, one being $\frac{1}{2}$ of the other; or one is $\frac{1}{2}$ of the total voltage, the other $\frac{1}{4}$. Hence between the middle ring and either outside ring the E.M.F. is

$$\sqrt{(\tfrac{1}{2})^2 + (\tfrac{1}{4})^2} = 0.56 \text{ of the main E.M.F.}$$

Therefore the formula for this is

$$E = \frac{1.12\, \phi n v p k}{10^8}. \tag{99}$$

n is the number of surface conductors in the main winding.

If the main E.M.F. is 1040 volts, the E.M.F. between the middle ring and either outside one is $0.56 \times 1040 = 580$ volts. It is recommended that the transformers for the induction motors be 9 to 1 or 4 to 1, instead of 10 to 1 or 5 to 1, so as to give a higher voltage at the motor terminals to allow for the inductive drop in the motors. If the transformers are connected as indicated, the three secondary voltages are approximately equal.

Example. — A *two-phase* ring armature with independent circuits is to be constructed for 520 volts at a frequency of 125. How many turns of wire are necessary for each phase when the flux from each pole is 10^6 maxwells?

Solution. — Since $E = \dfrac{1.11\, \phi n f}{10^8}$,

$$n = \frac{E \times 10^8}{1.11\, \phi f} = \frac{520 \times 10^8}{1.11 \times 10^6 \times 125} = 372 \text{ surface wires.}$$

Since each phase contains one-half of the total, each will require $\frac{1}{2}$ of $372 = 186$ conductors.

Example. — Suppose the same voltage is required in a *two-phase* machine for *three-wire* circuits. How many conductors would be necessary?

Solution. — Since the machine voltage, that is, the E.M.F. between either outside wire and the common middle wire, must be 520 volts, the same as before, each phase winding must give 520 volts, as before. Hence the same number of turns is required as in the last example. But the voltage measured between the two outside mains will be

$$\sqrt{520^2 \times 2} = \frac{1.57\,\phi n f}{10^8} = 735 \text{ volts.}$$

Example. — Let it be required to wind the armature for *three-phase* currents, *delta*, or *mesh* winding. How many turns will be required in each phase? Also for a line current of 20 amperes primary, how many amperes will flow in the armature winding?

Solution. — For 3-phase mesh

$$E = \frac{0.74\,\phi n f}{10^8};$$

whence

$$n = \frac{E \times 10^8}{0.74\,\phi f} = \frac{520 \times 10^8}{0.74 \times 10^6 \times 125} = 558.$$

Therefore each phase requires $\frac{1}{3}$ of $558 = 186$ wires. Also armature current = line current $\div \sqrt{3}$. Then

$$I' = \frac{I}{\sqrt{3}} = \frac{20}{1.732} = 11.5 \text{ amperes.}$$

EXAMPLE. — A similar machine is to be wound 3-phase, *star* connected. How many turns are required in each phase?

SOLUTION. — Referring to the figure given, it is apparent that each phase must generate the required line E.M.F. $\div \sqrt{3}$; or $E' = \dfrac{E}{\sqrt{3}} = \dfrac{520}{1.732} = 300$ volts.

Hence
$$n = \frac{300 \times 10^8}{0.74 \times 10^6 \times 125} = 324,$$

and
$$\tfrac{1}{3} n = 108 \text{ wires in each phase.}$$

EXAMPLE. — How many turns must be put in the main winding of a *monocyclic* generator for an E.M.F. of 2080 volts, assuming ϕ to be 2×10^6 maxwells and $f = 125$ cycles; also how many in the *teaser* winding so that the primary E.M.F. between the outside mains and the teaser line shall be 1100 volts?

SOLUTION. — From (98) the main E.M.F. is

$$E = \frac{2.22\, \phi n f}{10^8};$$

hence
$$n = \frac{E \times 10^8}{2.22\, \phi f} = \frac{2080 \times 10^8}{4.44 \times 10^6 \times 125} = 374.$$

Also, since the primary voltage for motors is to be 1100, the E.M.F. for teaser coils must be

$$E_t = \sqrt{1100^2 - \left(\frac{2080}{2}\right)^2} = 358 \text{ volts}$$

Therefore
$$n_t = \frac{358 \times 10^8}{2.22 \times 2 \times 10^6 \times 125} = 64 \text{ wires.}$$

48. **Original Problems.** — 1. What flux will a two-pole direct current drum type machine be required to supply for 220-volt lamps, say a 240-volt machine running at 1500 r.p.m. when there are to be 100 coils of 2 turns each on the armature? $\phi = 2.4 \times 10^6$ maxwells.

2. The following data are taken from a small Westinghouse bipolar incandescent dynamo. Armature body $8\frac{1}{4}$ inches $\times$ 8 inches, 100 grooves, two wires per groove. Normal speed 2100 r.p.m. It is also determined that the armature flux is 1,671,000 maxwells at full load. What should be the voltage of the machine under load when running at normal speed? $E = 117$ volts.

3. A T.–H. incandescent dynamo, motor type 3 D, capacity 3000 watts, furnishes the following data. There are 64 coils on the armature, 3 turns per coil; speed 2400 r.p.m., E.M.F. at $20\frac{1}{2}$ amperes, 116 volts. What should be the armature flux under the conditions of load named? $\phi = 755{,}208$ maxwells.

4. How many maxwells are due to the compounding when the above machine tests 110 volts on open circuit? Approximately, $\phi_c = 39{,}062$ maxwells.

5. The pulley on the machine mentioned in problem 2 has a diameter of 6 inches. At what speed must the machine run and how large a pulley will be required to give 120 volts, when the engine to which it is belted has a pulley 36 inches in diameter running at 350 r.p.m.?

Speed = 2154 r.p.m.
Pulley = 5.85″ diameter.

6. Suppose the armature in problem 3 is to be rewound for 125 volts, using the same fields but changing the commutator if necessary and running at the same speed as before. How many turns per coil will be necessary?

$n = 414$ wires $= 208$ turns.

Hence use 52 segments on commutator, and wind 4 turns to each coil.

7. The following data are known about a certain Westinghouse pony alternator. Speed 2000 r.p.m. E.M.F. 50 volts. There are 8 poles and 6 turns per coil on the armature. If we may assume the average constants to apply, what must be the flux from each pole? $\phi = 176,000$ maxwells.

8. An 8-pole alternator is rated at 1500 r.p.m. There are 12 turns per coil. If an estimate places the flux at 2,440,000 maxwells, what voltage should the machine give?

$E = 1040$ volts.

9. What will be the size of armature in the machine in problem 8 if $\frac{1}{10}$ inch is allowed for clearance, and the pole width is one half the pitch, not considering any leakage, assuming $B = 10,000$ gausses?

Length $= 9'' = 22.8$ cms.
Pole width $= 4.2'' = 10.7$ cms.
Pitch circle $= (8 \times 10.7) \times 2 = 171.2$ cms.
Diameter $= 171.2 \div \pi = 54.5$ cms.
Armature diameter $= 54$ cms.

10. How many poles will be required in an alternator for a voltage of 1040, surface conductors 200, flux 2,342,000 maxwells, speed 1200? Find frequency. $p = 5$ pairs.

$f = 100.$

11. What flux will be required by a drum armature wound for 6 poles, direct current, 240 volts, 1200 r.p.m., multiple connected, $n = 360$? $\phi = 3,333,000$ maxwells.

12. What E.M.F. would this last machine give if the armature circuits were series connected?

$E \doteq 720$ volts.

13. A 4-pole direct current dynamo is built for 120 volts under load. How many wires must be connected on the drum for a speed of 1200 r.p.m., multiple circuit armature, flux 3×10^6 maxwells? $n = 200$ conductors.

14. A half horse-power motor, 4 poles, is to be built to run at 2400 r.p.m., series connected. It is calculated that the polar faces are to be $1'' \times 4''$. The E.M.F. is to be 112 volts. A density of flux in the air gaps is assumed to be 10,000 gausses. How many turns per coil must be wound on the drum, for a 24-coil armature?

$n = 560$ wires $= 24$ per coil.

15. What flux must be sent into a 2-phase armature having 200 wires in each phase for an E.M.F. of 1040 volts, the machine being 12-pole and running at 900 r.p.m., connected for 4-wire circuits?

$\phi = 2.6 \times 10^6$ maxwells.

16. What would be the required magnetization for a voltage of 520 when the windings are connected for a two-phase, 3-wire circuit, other conditions the same as in 15? Also what is the frequency? What speed would give a frequency of 120? $\phi = 1.3 \times 10^6$ maxwells.

$f = 90$.

$v = 1200$ r.p.m. for 120 cycles.

17. How many turns must be wound in each phase of a two-phase alternator having ten poles, for 2080 volts, for a frequency of 60 and an estimated flux of 12×10^6 maxwells, circuits to be 3-wire? What is the speed?

n, each phase, = 130.
Speed = 720 r.p.m.

18. How much current must each phase winding be capable of carrying in problems 16 and 17, when it is desired that the wattmeter in the circuit read 50 K.W. full load in each case?

Effective line current in 16 = 96 amperes.
Total = 192.
Effective line current in 17 = 24 amperes.
Total = 48.

19. What is the E.M.F. between the outside wires in the machine described in problem 17?

$E = 2941.5$ volts.

20. What should be the E.M.F. of the following 3-phase, star connected alternator? Normal speed 1000 r.p.m., 12 poles, 560 conductors each phase, flux from each pole 804,000 maxwells. Also what is the frequency?

E.M.F. = 1729 volts.
$f = 100$.

21. A 3-phase mesh connected alternator having ten poles, running at 720 r.p.m., flux 7,600,000 maxwells, has a capacity of 250 K.W. What is the brush E.M.F., and the current in each phase winding, the total surface conductors being 600? $E = 2025$ volts.
$I = 41$ amperes per phase.

22. Let the armature in 21 be star connected. How will the E.M.F. and current differ from those just obtained?

E.M.F. = 3502 volts.
I = 41 amperes per phase.

23. Find the flux and the current per coil for a 3-phase machine, first when star connected, second when mesh connected, having given the following data. Surface conductors 864, poles 8, speed 900 r.p.m., brush E.M.F. 2500 volts, capacity 120 K.W.

Star, $\phi = 3.7 \times 10^6$ maxwells.
I = 28 amperes.
Mesh, $\phi = 6.5 \times 10^6$ maxwells.
I = 16 amperes.

24. How many turns must be put in the main winding of a monocyclic generator, $f = 60$, $p = 5$, $\phi = 5 \times 10^6$, $E = 1080$, capacity = 150 K.W.? Also if the teaser winding has just $\frac{1}{4}$ as many turns, how many volts should be measured between the main and teaser line wires? What is the effective current and the speed?

n = 10 coils, 8 turns each.
E.M.F. for motors = 605 volts.
I = 140 amp.; speed 720 r.p.m.

25. What flux must be provided for a monocyclic machine for 2160 volts, 10 poles, $f = 60$, main winding = 200 conductors? What is the current capacity of the wire when the machine is rated at 200 K.W? Also what should be the speed?

ϕ = 8,108,100 maxwells.
I = 92.6 amperes.
v = 720 r.p.m.

X.

CALCULATION OF FIELDS.

49. **Two-Pole Continuous Current Machines.** — We are now to apply to dynamos the principles of magnetic circuits which were worked out in a previous chapter. However, there are some practical details in connection with dynamo fields which it will be well to keep in mind in determining magnetomotive forces. The following examples will show that there are several parts in the magnetic circuit, — field cores, yoke, two air gaps, and armature core. Account must be taken of the flux density, length and cross section of each part. From these reluctances are found which multiplied by the total flux in each part gives the M.M.F. required for each portion of the circuit.

EXAMPLE. — A bipolar, drum armature, direct current machine requires 6×10^6 maxwells of flux. The armature core is $8'' \times 12''$. The air gaps including clearance and copper and insulation are each 1 centimeter in length. The poles are separated above and below the armature 4 inches. The field cores are $11''$ in diameter and $15''$ long. Polar heads, that is, the portion above the winding, average 8 inches in length of flux. The yoke between core centers is $20''$ and has a cross section of 100 square inches. The leakage coefficient is assumed to be 1.3. Determine the separate reluctances, the

M.M.F. in gilberts for each, the ampere-turns, and depth of winding and length of wire for an exciting current of 10 amperes. The armature is Norway iron, the rest is steel. See Fig. 16.* Check results by curves, p. 229.

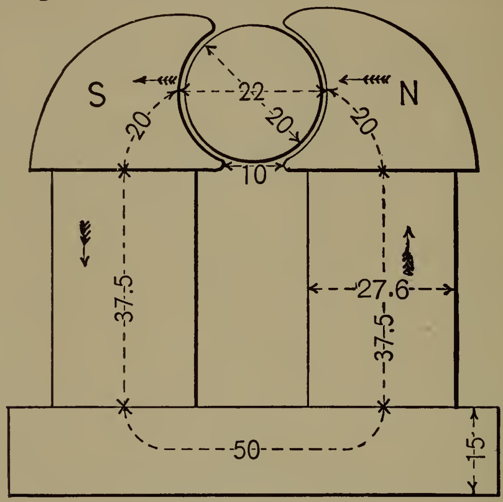


Fig. 16.

Armature: Diameter, $l = 8'' = 20$ cm.

Area, $A = 20 \times 30 = 600$ sq. cm.

Induction, $B_a = 6 \times 10^6 \div 600$

$= 10{,}000$ gausses.

$$\text{Reluctivity, } k = \frac{0.0001}{1 - 0.000059 \times 10^4}$$

$= 0.000244$ oersted per c c.

$$\text{Reluctance, } \mathcal{R} = 0.000244 \times \frac{20}{600}$$

$= 0.000008$ oersted.

M.M.F. $= 6 \times 10^6 \times 0.000008 = 48$ gilberts.

$$\text{Ampere-turns, } nI = \frac{48}{1.256} = 38.$$

* See also Example 4, p. 222.

Air gaps: Length, $l = 2$ cm.

$$\text{Area, } A = \frac{[22\pi - (10 \times 2)] \times 30}{2}$$

$$= 736.72 \text{ sq. cm.}$$

Reluctivity, $k = 1$.

Reluctance, $\mathcal{R} = 1 \times \dfrac{2}{736.72} = 0.00271$ oersted.

M.M.F. $= 6 \times 10^6 \times 0.00271$
$= 16{,}260$ gilberts.

Ampere-turns, $nI = 16{,}260 \div 1.256$
$= 13{,}000$.

Poles: Length, $l = 20$ cm.

Area, $A = 736.72$ sq. cm., — same as air gap.

Induction, $B_p = 6 \times 10^6 \div 736.72 = 8{,}144$ gausses.

$$\text{Reluctivity, } k = \frac{0.00045}{1 - 0.000051 \times 8{,}144}$$

$= 0.00077$ oersted per cc.

$$\text{Reluctance, } \mathcal{R} = 0.00077 \times \frac{20}{736.72} \times 2$$

$$= 0.000418 \text{ oersted.}$$

M.M.F. $= 10^6 \times 6 \times 0.0000418$
$= 250$ gilberts.

Ampere-turns, $nI = 250 \div 1.256 = 200$.

Cores: Length, $l = 15'' = 37.5$ cm.

Area, $A = \overline{27.6}^2 \times 0.7854 = 600$ sq. cm.

Induction, $B_c = (6 \times 10^6 \times 1.3) \div 600$
$= 13{,}000$ gausses.

$$\text{Reluctivity, } k = \frac{0.00045}{1 - 0.000051 \times 13 \times 10^3}$$
$= 0.00133$ oersted per cc.

$$\text{Reluctance, } \mathcal{R} = 0.00133 \times \frac{37.5}{600} \times 2$$
$= 0.000166$ oersted.

M.M.F. $= 6 \times 10^6 \times 1.3 \times 0.000166$
$= 1295$ gilberts.

Ampere-turns, $nI = 1295 \div 1.256$
$= 1031.$

Yoke: Length, $l = 50$ cm.

Area, $A = 100$ sq. in. $= 645$ sq. cm.

Induction, $B_y = 6 \times 10^6 \times 1.3 \div 645$
$= 12{,}093$ gausses.

$$\text{Reluctivity, } k = \frac{0.00045}{1 - 0.000051 \times 12{,}093}$$
$= 0.00117$ oersted per cc.

$$\text{Reluctance, } \mathcal{R} = 0.00117 \times \frac{50}{645}$$
$= 0.00008$ oersted.

M.M.F. $= 10^6 \times 6 \times 1.3 \times 0.00008$
$= 625$ gilberts.

Ampere-turns, $nI = 625 \div 1.256$
$= 500.$

Hence the total ampere-turns $= 38 + 13{,}000 + 200 + 1031 + 500 = 14{,}769 = 7385$ on each core. For 10-ampere

current, this requires $7385 \div 10 = 738$ turns on each core. About 1000 circular mils per ampere should be allowed in field windings. Hence 10,000 circular mils is the necessary cross section of the wire, corresponding to No. 10 B. & S., whose diameter is 102 mils bare, or say 120 mils double cotton covered. Therefore allowing $\frac{1}{4}$ inch at each end of the core for collar and insulation, there will be in one layer $[(15 - \frac{1}{2}) \div 0.120] = 120$ turns. This requires a depth of winding $= 738 \div 120 = 7$ layers, nearly, allowing for insulation on cores before wire is put on. Only about $6\frac{1}{2}$ are needed for the excitation. $120 \times 6\frac{1}{2} = 780$ turns to the core. The cores may have been shortened slightly, or a little heavier insulation put on against the collars so as to require just seven layers. Depth of winding $= 7 \times 0.120 = 0.84$ inch. Total addition to the diameter of the field $= 0.84 \times 2 = 1.68$ inches. $\frac{1.68}{11} = \frac{1}{7}$ of core diameter.

The average diameter of the finished field is $11 + .84 = 11.84$ inches. Circumference of the mean turn $= 11.84 \times \pi = 37.2$ inches.

Total length of wire necessary for excitation $= \dfrac{37.2 \times 1476}{12}$

$$= 4575 \text{ feet.}$$

$$R_f = \frac{4575 \times 10.8}{10{,}382} = 4.7 \text{ ohms cold.}$$

$$R_f \text{ hot} = 4.7 + 20\% \text{ of } 4.7 = 5.64 \text{ ohms.}$$

Wiring table gives for No. 10 wire 1.034 ohms per 1000 ft. Hence

$$R_f (\text{from table}) = 1.034 \times 4.575 = 4.7 \text{ ohms cold.}$$

50. **Consequent Pole Type.** — Example. — Let it be required to determine the winding of a *consequent pole* field for the armature in the last example, the speed, voltage, flux, etc., to remain the same, and the dimensions of the field parts to be as follows: diameter of the cores (four) 21.5 cm.; average length of poles for flux lines 18 cm.; air gap 1 cm.; separation of pole tips 10 cm.; length of cores 20 cm.; height of yokes 65 cm.; total length of machine 90 cm. The coefficient of leakage for this type is taken at 1.7. Check by curves, p. 229.

Solution. — Since there are two magnetic circuits in parallel in this type it will be well to calculate each circuit separately, each side furnishing one-half the required flux. See Fig. 17.

Armature: Diameter, $l = 20$ cm.

Area, $A = 20 \times 30 = 600$ sq. cm. $= 300$ each half.

Induction, $B_a = 3 \times 10^6 \div 300$
$= 10{,}000$ gausses.

$$\text{Reluctivity, } k = \frac{0.0001}{1 - 0.000059 \times 10^4}$$
$= 0.000244$ oersted per cc.

$$\text{Reluctance, } \mathcal{R} = 0.000244 \times \frac{20}{300}$$
$= 0.000016$ oersted.

M.M.F. $= 3 \times 10^6 \times 0.000016$
$= 48$ gilberts.

Ampere-turns, $nI = 48 \div 1.256 = 38$.

Air gaps: Length, $l = 2$ cm.

Area, $A = 736.72 \div 2 = 368.36$ for half the flux.

Reluctivity, $k = 1$.

Reluctance, $\mathcal{R} = 1 \times \dfrac{2}{368.36} = 0.00542$ oersted.

M.M.F. $= 3 \times 10^6 \times 0.00542 = 16{,}260$ gilberts.

Ampere-turns, $nI = 16{,}260 \div 1.256 = 13{,}000$.

Poles: Length, $l = 18$ cm.

Area, $A = 736.72 \div 2 = 368.36$ sq. cm.

Induction, $B_p = 3 \times 10^6 \div 368.36$

$= 8144$ gausses.

Reluctivity, $k = \dfrac{0.00045}{1 - 0.000051 \times 8144}$

$= 0.00077$ oersted per cc.

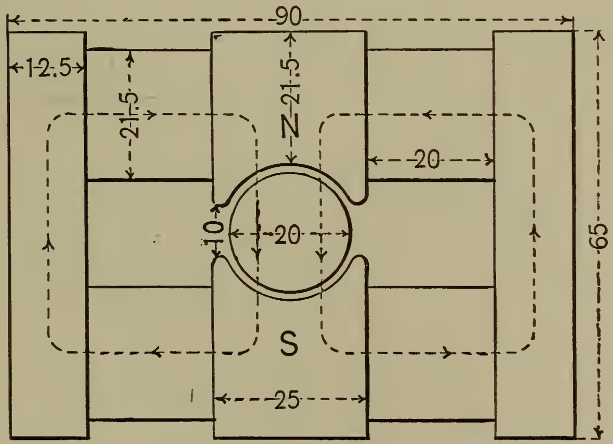


Fig. 17.

Reluctance, $\mathcal{R} = 0.00077 \times \dfrac{18}{368.36}$

$= 0.000038$ oersted.

M.M.F. $= 2 \times 3 \times 10^6 \times 0.000038$

$= 228$ gilberts.

Ampere-turns, $nI = 228 \div 1.256 = 182$.

Cores: Length, $l = 20$ cm.

Area, $A = \overline{21.5}^2 \times 0.7854 = 363$ sq. cm.

Induction, $B_c = 3 \times 10^6 \times 1.7 \div 363$

$= 14{,}060$ gausses.

$$\text{Reluctivity, } k = \frac{0.00045}{1 - 0.000051 \times 14{,}060}$$

$= 0.00159$ oersted per cc.

$$\text{Reluctance, } \mathcal{R} = 0.00159 \times \frac{20}{363} \times 2$$

$= 0.0001752$ oersted.

M.M.F. $= 3 \times 10^6 \times 1.7 \times 0.0001752$

$= 894$ gilberts.

Ampere-turns, $nI = 894 \div 1.256 = 712$.

Yoke: Length, $l = 42$ cm. for lines of force.

Area, $A = 12\frac{1}{2} \times 30 = 375$ sq. cm.

Induction, $B_y = 3 \times 10^6 \times 1.7 \div 375$

$= 13{,}600$ gausses.

$$\text{Reluctivity, } k = \frac{0.00045}{1 - 0.000051 \times 13{,}600}$$

$= 0.00115$ oersted.

$$\text{Reluctance, } \mathcal{R} = 0.00115 \times \frac{42}{375}$$

$= 0.000128$ oersted.

M.M.F. $= 3 \times 10^6 \times 1.7 \times 0.000128$

$= 652$ gilberts.

Ampere-turns, $nI = 652 \div 1.256 = 519$.

Hence the total ampere-turns for each circuit $= 38 + 13{,}000 + 182 + 712 + 519 = 14{,}451$. Total reluctance two paths in parallel, $0.005777 \div 2 = 0.002888$ oersted.

The total M.M.F. for each circuit $= 18{,}151$ gilberts.

The machine will require on each of the four cores $\frac{1}{2}$ of 14,451 = 7226 ampere-turns. If we connect all the field coils in series, the wire will be of the same size as in last example, namely No. 10 B. & S. = 120 mils d.c.c. Allowing 1 centimeter for collars and insulation on each core, we have 19 cm. for wire = 7.5 inches, nearly. In one layer there will be $7.5 \div 0.120 = 62$ turns which will require $722.6 \div 62 = 12$ layers approximately, making a depth of winding $0.120 \times 12 = 1.44$ inches $= 2 \times \frac{1.44}{8.6} = \frac{1}{3}$ of core diameter.

Mean length of one turn $= \frac{(8.6 + 1.44)\pi}{12} = 2.7$ feet.

Total length of No. 10 wire for fields $= (2.7 \times 744) \times 4 = 8035$ feet. Total weight (32 lbs. per 1000 feet, approximately) $= 8.035 \times 32 = 257$ lbs.

Resistance of field winding

$$R_f = \frac{8035 \times 10.8}{10{,}382} = 8.3 \text{ ohms cold};$$

$$8.3 + 20\% \text{ of } 8.3 = 9.96 \text{ ohms hot.}$$

Drop in field winding $= 9.96 \times 10 = 99.6$ volts.

Suppose now we wind the two halves for 5 amperes each and connect them in parallel. Each spool will require $7226 \div 5 = 1445$ turns of 5000 circular mils each cross section. This corresponds to No. 14 B. & S. = 64 mils bare, or about 80 mils d.c.c. In one layer there will be 100 turns, and 14 layers deep $= 14 \times 0.08 = 1.12$ inches $= \frac{1}{4}$ of core diameter. Mean length of turn $= \frac{(8.6 + 1.12)\pi}{12} = 2.5$ feet, requiring $1445 \times 2.5 = 3612.5$

feet on each core, or 14,450 feet total length, having a resistance hot of 38 ohms. But since two halves are in parallel the field resistance will be $\frac{1}{4}$ of $38 = 9.5$ ohms and the drop $= 9.5 \times 10 = 95$ volts. The weight of No. 14 wire will be $14.45 \times 13 = 190$ lbs., provided the assumptions about insulation are correct.

The radiating surface for the last case will be $(8.6 + 2.24)\,\pi \times 8 = 272$ sq. in. on each coil, or 1088 sq. in. total. Total watts lost $= 95 \times 10 = 950$ watts, making about 1 watt per square inch.

51. The Multipolar Type. — The preceding armature core is to be wound for a four-pole field and series connected, thus requiring 3×10^6 lines of force from each pole, instead of 6×10^6. The following are the field dimensions: mean circumference of pole ring 190 cm.; width 6.5 cm.; length of cores 16 cm.; width of cores 8.6 cm.; coefficient of leakage for this type 1.3. The field ring and poles are steel. Form shown in Fig. 18.

Armature: Length (for flux), $l = 20$ cm.

Area, $A = 10 \times 30 = 300$ sq. cm.

$$\text{Induction, } B_a = \frac{3 \times 10^6}{2} \div 300$$

$$= 5000 \text{ gausses.}$$

$$\text{Reluctivity, } k = \frac{0.0001}{1 - 0.000059 \times 5000}$$

$$= 0.00014 \text{ oersted per cc.}$$

$$\text{Reluctance, } \mathcal{R} = 0.00014 \times \frac{20}{300}$$

$$= 0.0000093 \text{ oersted.}$$

$$\text{M.M.F.} = \frac{3 \times 10^6}{2} \times 0.0000093$$

$$= 13.95 \text{ gilberts.}$$

$nI = 13.95 \div 1.256 = 11$ ampere-turns.

Air gaps: Length, $l = 2$ cm.

$$\text{Area, } A = \frac{8.6 \times 30}{2} = 129 \text{ sq. cm.}$$

$$\text{Reluctance, } \mathcal{R} = 1 \times \frac{2}{129} = 0.0155 \text{ oersted.}$$

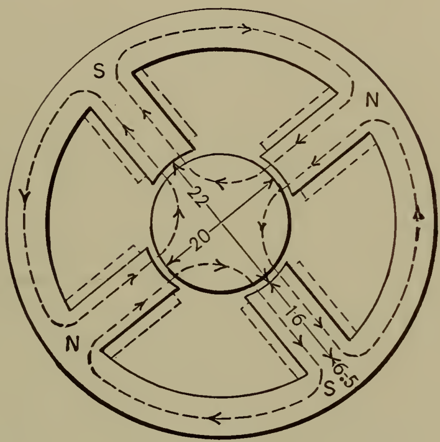


Fig. 18.

$$\text{M.M.F.} = \frac{3 \times 10^6}{2} \times 0.0155$$

$$= 23{,}250 \text{ gilberts.}$$

$nI = 23{,}250 \div 1.256 = 18{,}500$ amp.-turns.

Poles: Length, $l = 16$ cm.

$$\text{Area, } A = \frac{8.6}{2} \times 30 = 129 \text{ sq. cm.}$$

$$\text{Induction, } B_p = \frac{3 \times 10^6}{2} \times 1.3 \div 129$$

$$= 15{,}120 \text{ gausses.}$$

$$\text{Reluctivity, } k = \frac{0.00045}{1 - 0.000051 \times 15{,}120}$$
$$= 0.0016 \text{ oersted per cc.}$$

$$\text{Reluctance, } \mathcal{R} = 0.0016 \times \frac{16}{129} \times 2$$
$$= 0.0004 \text{ oersted.}$$

$$\text{M.M.F.} = \frac{3 \times 10^6}{2} \times 1.3 \times 0.0004$$
$$= 780 \text{ gilberts.}$$

$$nI = 780 \div 1.256 = 621 \text{ ampere-turns.}$$

Yoke: Length, $l = 190 \div 4 = 47.5$ cm.

Area, $A = 6.5 \times 30 = 195.0$ sq. cm.

$$\text{Induction, } B_y = \frac{3 \times 10^6 \times 1.3}{2} \div 195$$
$$= 10{,}000 \text{ gausses.}$$

$$\text{Reluctivity, } k = \frac{0.00045}{1 - 0.000051 \times 10{,}000}$$
$$= 0.0009 \text{ oersted per cc.}$$

$$\text{Reluctance, } \mathcal{R} = 0.0009 \times \frac{47.5}{195}$$
$$= 0.00022 \text{ oersted.}$$

$$\text{M.M.F.} = \frac{3 \times 10^6 \times 1.3}{2} \times 0.00022$$
$$= 429 \text{ gilberts.}$$

$$nI = 429 \div 1.256 = 341 \text{ ampere-turns.}$$

As before, the coefficient of leakage is applied to the yoke as well as to the cores, since it is uncertain just what the leakage paths are and our error in making the leakage factor apply to the yoke will be on the safe side.

From our data the reluctance of a single magnetic circuit will be

$$\begin{aligned}\mathcal{R} &= 0.0000093 + 0.0155 + 0.0004 + 0.00022 \\ &= 0.0161 \text{ oersted.}\end{aligned}$$

Hence the combined reluctance of two parallel paths will be

$$0.0161 \div 2 = 0.00806 \text{ oersted.}$$

Also the combined M.M.F. of two parallel paths will be

$$\begin{aligned}\text{M.M.F.} &= 13.95 + 23{,}250 + 780 + 429 \\ &= 24{,}473 \text{ gilberts.}\end{aligned}$$

$$nI = 24{,}473 \div 1.256 = 19{,}485 \text{ ampere-turns.}$$

This requires 9742 ampere-turns on each core. If we take as exciting current 5 amperes, the number of turns will be $9742 \div 5 = 1944$ on each core, requiring 5000 circular mils section, corresponding to No. 14 wire = about 80 mils diameter d.c.c.

The length of core winding = 16 cm. = 6300 mils. Hence there will be $6300 \div 80 = 80$ turns in one layer, thus requiring $1944 \div 80 = 24$ layers deep $= 24 \times 0.080 = 1.9$ inches. The mean length per turn $= 3\frac{1}{6}$ feet, making the total length required on the fields,

$$l = 1944 \times 3\tfrac{1}{6} \times 4 = 24{,}624 \text{ feet.}$$

The resistance for the fields connected in series will be

$$24.63 \times 2.58 = 63.6 \text{ ohms cold.}$$

Since the resistance of copper increases about 1 per cent for each 2° C. rise above the air, and allowing 40° rise, the resistance will become

$$R \text{ hot} = 63.6 + \left[(1\% \text{ of } 63.6) \times \frac{40}{2}\right] = 76.3 \text{ ohms.}$$

The voltage required for excitation is

$$E = 76.3 \times 5 = 381.5 \text{ volts.}$$

If we connect the fields so that two will be excited in series and two in parallel, the total resistance becomes $R = \frac{19.1 \times 2}{2} = 19.1$ ohms hot. The total exciting current must now be 10 amperes, and the voltage for excitation becomes

$$E = 19.1 \times 10 = 191 \text{ volts.}$$

The weight of wire will be 325 to 350 lbs.

Fields for alternators and direct current machines of any number of poles are determined as in the last example.

It is impossible to be absolutely exact in these preliminary calculations, since it is not definitely known just what the leakage paths will be and just what the behavior of the particular quality of iron will be under test, also the effect of joints, etc. Manufacturers, of course, must govern their final design by tests of a finished machine as well as by tests of the material used.

For compound wound machines, if the E.M.F. at no load and also at full load are known, or if left for the designer to decide, the series winding is determined so as to supply flux through the known reluctances to make up the difference of these two E.M.F's plus the additional leakage. Also if the armature resistance and maximum current are given, the armature drop due to the load is $E_a = IR_a$. Hence if the compounding is only for armature drop, we can determine the series winding to supply the extra pressure necessary.

All armature bodies are laminated, so that the actual area is reduced 10% to 20%. But since the armature reluctance is such a very small part of the total, it will not be necessary to correct for this reduction of area due to lamination.

In cases where the general dimensions of a dynamo are given, a good plan is to lay down the machine to scale on a drawing board, then take from the drawing all distances and areas needed in calculating reluctances or $\frac{nI}{l}$.

52. **Original Problems.** — 1. Determine the shunt winding, depth, resistance, etc., for the following machine, two-pole, direct current. Armature body $8\frac{1}{4}'' \times 8''$, 100 grooves, two wires per groove, resistance 0.06 ohm; $I = 30$ amperes; speed = 2025 r.p.m.; E.M.F. no load 118 volts, full load 120 volts. Field bore $8\frac{1}{32}$ inches; core length, single, 6 inches; area 63.68 square inches; separation at tips $4\frac{1}{4}$ inches; mean length of lines of force in each pole 17 inches; pole width 8 inches. Assume wrought iron throughout. Also calculate the series turns for compounding.

Shunt coil 3350 turns.
Wire No. 21 = 0.0354″ covered.
$R_f = 120$ ohms.
Series coil 17 turns of No. 3 = 0.283″.
Resistance series 0.013 ohm.

2. What will be the required number of series and shunt turns for the following two-pole, direct current machine? Armature body $8'' \times 6''$; 64 coils, 3 turns each; resistance 0.85 ohm; speed 2400 r.p.m.; E.M.F. no

load 110 volts; full load 116 volts. Field cores each 6 inches long, $5\frac{1}{2}$ inches diameter; pole heads from top of cores to center line of armature 15 inches; area of pole heads $8 \times 3 = 24$ square inches; tips $2\frac{1}{4}$ inches apart. Yoke length for lines 13 inches; width 8 inches; depth $4\frac{1}{2}$ inches. Air gap including the wire on the armature 0.3 inch. The yoke is of cast iron; armature Norway; cores and poles dynamo wrought iron. Leakage factor is 1.25.

Shunt 4850 turns No. 20 wire.
Series 46 turns No. 8 wire.

3. A small 50-volt, 8-pole alternator has an armature core $5\frac{1}{2}$ inches diameter and 6 inches long, and is wound with 8 coils, 6 turns each, No. 14 wire. The external diameter of the field ring is 13.5 inches, internal 11 inches; width 5 inches. The clearance is 0.3 centimeter. The poles are $1'' \times 6''$, and $2\frac{1}{2}$ inches long. Speed of armature 2000 r.p.m. All parts are made of wrought iron. Allowing about 600 circular mils per ampere in fields, find the number of turns and depth of wire on each core.

Field 492 turns, 12 layers of No. 19.
Total, 8 cores = 3936 turns.
Depth on each core 0.6 inch.
$R_f = 45$ ohms if connected in series.

4. A consequent pole machine similar to the one previously worked out has the following data taken from it: Armature body $8'' \times 8''$; slotted for 200 wires, 2 per groove; speed 2400; E.M.F. 116 volts; core laminated Norway iron; field cores wrought iron, 6 inches long, 4

inches diameter; yokes cast iron, 3 inches wide, 12 inches between core centers; poles cast iron, 8 inches between cores, 5 inches perpendicularly, tips separated 4 inches. Air gaps are 0.15 cm. each. Determine the turns necessary to wind the machine as a shunt motor, allowing 2 amperes for exciting current. Also calculate the length of wire, its depth, resistance and weight.

$n = 810$ turns each core; 3240 total.
Length of wire 3825 feet of No. 20.
$R_f = 43$ ohms, allowing 20% for rheostat.
Depth with insulation 0.4″.
Weight 12 lbs.

5. The drum armature for a 25-K.W. machine, 125 volts, is 11.5 inches long, 11.5 inches diameter. $B_a =$ 10,000 gausses; speed 1000 r.p.m. There are 112 armature conductors. Fields are Edison type, wrought iron. Core length, exclusive of collars, etc., 16.6 inches; diameter $11\frac{5}{8}$ inches. Height from top of yoke to lower end of poles 42 inches. Total length of yoke 34 inches. Air gaps, including winding, insulation, etc., on the armature, 0.05 inch. Pole tips 3 inches apart. Draw a diagram and determine the length of flux paths. Also determine the field winding for an exciting current equal to 2.5 per cent of output and for a leakage factor for this type of 1.6.

$n = 1750$ turns, each core.
Wire No. 13; length 11,060 feet.
$R_f = 22.18$ ohms.

NOTE. — A smaller size of wire may have been used, say No. 14 wire, without undue heating. The No. 13 wire was obtained by taking the resistance which the winding at 125 volts must have to allow just the required

exciting current to flow, and assuming at the outset a depth of winding about $\frac{1}{4}$ of core radius in order to obtain the length. Whence $d^2 = \frac{l \times 10.8}{R} = 5000$ circular mils, approximately. This may be used as a satisfactory check on any method of determining the field winding, or as the primary method, itself, to be checked by others.

6. The following data for an Edison-Hopkinson machine are taken from Thompson's "Dynamo-Electric Machinery": * normal output 320 amperes at 105 volts; speed of armature 750 r.p.m., diameter of armature core 25.4 cm., length 50.8 cm., carrying 40 turns, 2 layers deep; wire 1.75 mm. diameter. Magnet cores nearly rectangular, rounded corners; all material wrought iron; length of magnet limb 45.7 cm.; breadth 22.1 cm.; width parallel to shaft 44.45 cm. Length of yoke 61.6 cm.; width 48.3 cm.; depth 23.2 cm. Diameter pole bore 27.5 cm.; depth of pole piece 25.4 cm.; width parallel to shaft 48.3 cm.; width between pole pieces 12.7 cm. The area of iron in the armature is 810 sq. cm. The angle subtended by the pole face is 129°. Effective area of pole is 1600 sq. cm. The air gap is 1.5 cm. Design the shunt winding for 6 amperes exciting current.

$n = 3260$ turns, 11 layers on
each core, No. 12 wire.
$R_f = 17$ ohms, approximately.

7. An 8-pole direct current machine has an armature 31.1 cm. × 13 cm. cross section, Gramme ring type, 0.92 of area effective, on account of insulation of the ribbon of

* *Dynamo-Electric Machinery*, page 413.

armature. Speed is 1722 r.p.m. The armature carries 768 conductors. $B_a = 9,500$ gausses. External diameter of armature ring is 147.3 cm.; finished, 149.3 cm. Internal diameter of ring, 121.3 cm.; finished, 119 cm. The external diameter of pole ring is 231 cm.; internal diameter, 218.4 cm.; width parallel to shaft, 34.3 cm.; width of cores, 29.5 cm. The poles widen at the air gap so the latter is $41 \times 34 = 1400$ sq. cm. The armature coils are parallel connected. Determine the E.M.F., the reluctance and field winding, the armature being soft iron, the fields all cast steel. If the exciting current be 10 amperes, how many turns and how deep on each core?

$E = 155.7$ volts.
$nI = 7333$ each spool.
$n = 734$ turns each.
Depth $= 1.4''$.
$\mathcal{R}_f = 5.214 \times 10^{-3}$ oersted.

8. An iron-clad armature for a multipolar dynamo has been designed so that it requires in its teeth from each air gap 20.4×10^6 maxwells of flux. The armature is of sheet iron 14 inches long, cross section of teeth under pole 151.5 sq. in.; length of teeth 1.75 inches; armature cross section 124.3 sq. in. Air gap, length 0.25 inch; area 538 sq. in. Field poles, cross section 237.2 sq. in.; length 16 inches, cast steel. Yoke, section 132 sq. in.; length 28.5 inches, cast steel. One coil on each pole will require how many ampere-turns each, leakage coefficient being assumed as small as 1.2?

$nI = 5850$ ampere-turns.

XI.

ELEMENTS OF DYNAMO DESIGN.

53. **General Considerations.** — There is such a diversity of elements upon which the judgment must depend in the proper design of machines of the same type, to say nothing of different types, that only those details which illustrate the general principles of dynamo design can be given in this chapter. The general theory of the dynamo must be understood and the exact character of all the material going into its construction. Observation and test of various types and sizes of practical machines must be made use of. Then the manufacturers must gather experience, not only from continuous tests of all material used, but from the performance and tests of the completed product. In fact, there are so many interdependent quantities which themselves depend on the conditions of service and exact quality of material, that an approximation to exactness is all that can be expected in a preliminary design.

54. **Useful Data.** — (1) The **output** of dynamos is approximately proportional to their weight, or to the cube of a linear dimension, if of the same type, since the larger must have reduced speed, increased efficiency, greater radiating surface, etc. Double the weight of copper and double the weight of iron should give double the output. The output will be about 6 watts per pound for continu-

ous current belted dynamos, and 8 watts per pound for direct driven multipolar machines.

(2) The **temperature** of armatures should generally not exceed about 50° C. above the air. Although the subject is somewhat indefinite, yet tests tend to show that the amount of external surface in an armature should be taken so that 1 sq. in. may be allowed for each 2 to $2\frac{1}{2}$ watts lost in the armature. Of this loss about 1 watt per sq. in. will be due to I^2R loss, and the rest to hysteresis and eddy currents.

For example, if the I^2R loss is 2% of 100 K.W. = 2000 watts, the external surface of the armature should be $\frac{2000}{1} = 2000$ sq. in. In designing armatures this method may be used for determining the size of armature. In any event it should be used as a check on any other method employed to obtain the required size of armature. If armatures are internally ventilated, a smaller area per watt can be used.

The fields should have somewhat larger surface per watt lost owing to the fact that the winding is deeper and there is no mechanical ventilation as in armatures. Some recommend 2 square inches per watt lost in field windings. This may be assumed as a safe limit, though in many cases only 1 square inch per watt is allowed.

In very small sizes of machines the temperature is permitted to go higher, since the same efficiency is not attempted as in large sizes. Often as low as 250 to 400 circular mils per ampere is used for the armature wires and 500 to 800 for the field windings. But as a working average for ordinary machines 600 circular mils per

ampere should be taken for armature conductors, and 800 to 1000 circular mils per ampere for the fields.

(3) The proper **I^2R losses** for armatures and fields may be arrived at from the following table derived from several standard modern machines of various sizes.

7. TABLE OF I^2R ARMATURE AND FIELD LOSSES.

SIZE.	ARMATURE LOSS %.	FIELD LOSS %.
100 K.W.	2.0	1.1
50 K.W.	2.2	1.6
25 K.W.	2.5	2.5
5 K.W.	4.0	4.5

(4) **Peripheral speeds** of armatures vary from about 3000 feet per minute as an average in ordinary sizes, to 5000 feet per minute or more in very large sizes such as Ferranti disk armatures.

(5) The **angle of span** of the poles in two pole dynamos varies from 120° to 145°. An educated judgment as to the proper separation of the pole tips so as to reduce leakage can be relied on in this particular. Poles may cover about three-fourths of armature surface. Say 0.6 to 0.75.

(6) The **armature laminæ** vary in thickness from 25 to 50 mils; say 30 mils as an average. Usually 10 per cent to 20 per cent may be allowed as a reduction in the armature cross section for insulation between laminæ and shaft space.

(7) For **armature proportions** we may take the ratio of the external to the internal diameter 10 to 8; sometimes 10 to 7 is used. For drum disks the ratio is about 10 to 3. These values are not rigid. Different values would

perhaps be assigned by different designers. The ratios of drum armature lengths to diameters vary considerably. But representing the length by l and the diameter by d, the ratios are most often $\frac{l}{d} = 1$, $\frac{l}{d} = 2$, or some ratio between these. For ring armatures the ratios vary from $\frac{l}{d} = \frac{1}{2}$ to $\frac{l}{d} = 2$.

(8) **Flux densities** in armatures for constant potential direct current machines vary between 10,000 gausses and 15,000 gausses. However, it is not often advisable to go beyond 12,000 gausses. For *arc machines* the density often reaches 18,000 gausses. Perhaps 16,000 gausses will represent the average flux density for such machines. For alternators the density is considerably less, varying from 6000 to 7000 gausses, while in coreless disks it is about 5000 gausses.

Air gap intensities vary from 3000 to 7000 gausses in direct current machines, and from 2500 to 5000 in alternators.

Field densities vary from 12,000 to 17,000 in both direct constant potential machines and alternators. It will be about 18,000 gausses in arc machines. These values are all for wrought iron. When cast iron fields are used 6000 to 8000 gausses is as high as permissible in any type.

(9) For **relative cross section** of field and armature Thompson gives the following ratios of field to armature; ring machines, wrought field, 1.66; cast field, 3. For

drum armatures, wrought field, 1.25; cast field, 2.3. These, however, are only tentative, being subject to considerable variation.

55. **Armature Magnetization.** — In the field calculation of Chapter X. no account was taken of armature reaction due to the magnetizing effect of the armature current. In the design of dynamos allowance must be made for this armature magnetization, otherwise the voltage will not be high enough, nor constant at that; for the larger the armature current the greater will be its effect in neutralizing and distorting the field.

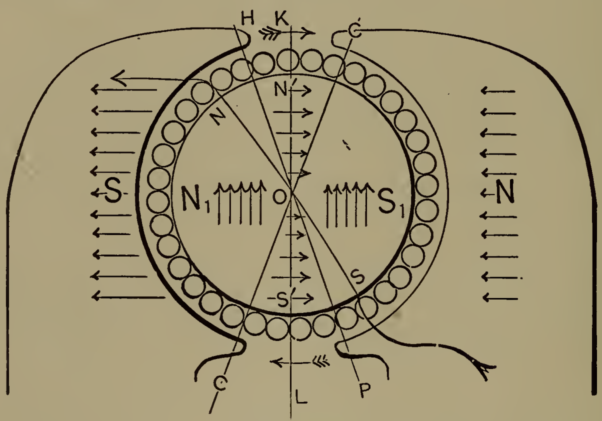


Fig. 19.

In most machines there are two effects of the current in the armature conductors. Referring to Fig. 19, the direction of the field magnetization is represented by the darts on the poles, and the two effects of armature current by the darts on the armature, showing the direction of the lines of force; those on the right completing their circuit through one half of the armature and the *N*-pole; of course, there is a similar circuit on the left.

The brushes stand in the plane, CC called the *commutating plane; HOC* is double the *angle* of *lead*, which is KOC; LK is the normal plane, and if there were no skewing effect due to armature reaction, it would be the plane containing the brushes. The turns in the angle HOC, four in this case, set up an armature magnetization directly opposed to that of the field. These turns are therefore called *back turns*. The rest of the armature turns, approximately those coming under the span of the poles, set up a magnetization whose lines tend to cross those due to the field, and whose poles in the armature tend to be at right angles to those induced in the armature by the field. These armature turns are called *cross turns*. Since there cannot exist two independent sets of poles in the armature core, there will be a resultant of the induced magnetization N_1S_1 and the cross magnetism $N'S'$, along the line NS diagonally between the components. The combined effect is apparently a pulling down of the lines in one pole and a pulling up in the other in the direction of revolution; and the resultant magnetism passes diagonally up through the armature core instead of horizontally as it would were there no armature magnetism.

The ampere-turns on the armature causing the *back* magnetization are

$$\text{Back ampere-turns} = n_b \times \tfrac{1}{2} I \qquad (100)$$

in which n_b is the number of conductors within the double angle of lead. In the figure $n_b = 4$. Also I is the whole current output of the machine.

The ampere-turns on the armature causing the *cross* magnetization are

$$\text{Cross ampere-turns} = n_c \times \tfrac{1}{2} I, \qquad (101)$$

in which n_c is the number of conductors on one-half the armature less n_b, the number in the double angle of lead. In other words n_c is the number counted within the angle COP or COH. I, as before, is the total armature current. We take $\frac{1}{2} I$, since each conductor carries but one-half the total current.

Suppose, as in the figure, that there are 14 cross turns and 20 amperes output. The back ampere-turns are $4 \times \frac{1}{2} \times 20 = 40$, and the cross ampere-turns are $14 \times \frac{1}{2} \times 20 = 140$.

If no other means of knowing what portion of the surface turns are *back* and what *cross*, an approximation may be made by taking the number between the pole tips as the back turns and those within the span of the pole as cross turns.

Evidently when the angle of lead is known the number of turns within double this angle is readily found. For example, let a be the angle of lead and n the whole number of armature turns; then

$$n_b = \frac{2\,a}{180^\circ} n = \frac{a}{90^\circ} n.$$

Let $$a = 15^\circ.$$

Then $$n_b = \frac{15\,n}{90} = \frac{n}{6}.$$

56. **Compounding.** — The *shunt field* winding is the only one effective when the machine is on open circuit; that is, when it is running at normal speed, but unloaded.

Therefore the regulation must be effected by compounding the dynamo; that is to say, by adding coils on the cores in *series* with the circuit and with the armature. These series coils must compensate for the drop in the armature resistance and in their own resistance; they must also compensate for the effects of the *back* and the *cross* magnetizations; they must in addition compensate for the effect of the increased saturation of the fields and armature when the machine is loaded. Furthermore in the case of *over compounding*, still more series turns must be added to compensate for a certain amount of line loss.

57. **Useful Formula.** — *Volts drop:* let R_a = the armature resistance, I the armature current, E_a = the volts drop in the armature. Then

$$E_a = R_a I. \tag{102}$$

If R_s is the field resistance, I_s = the field current, E_s = the field drop, then

$$E_s = R_s I_s. \tag{103}$$

Since the shunt winding and the external circuit are in parallel, E_s is the same as the external, or line E.M.F. The current in the field winding is the total current times the shunt field loss per cent. This varies somewhat with the size of the machine; see Sec. 54. But in ordinary sizes it will be about 2.5%.

The *series field drop* will be

$$E_f = R_f I_e. \tag{104}$$

R_f is the resistance of the series winding, I_e is the current

carried to the external circuit; it is the total I less the shunt field exciting current I_s. In other words,

$$I = I_e + I_s. \tag{105}$$

It will make but little difference in the temperature of the armature to design its coils for the external current, making it carry in operation the extra amperes necessary for the shunt excitation.

The *E.M.F. to be generated in the armature coils* is

$$E = E_a + E_f + E_e. \tag{106}$$

Strictly speaking, E_a includes the drop due to *cross* and *back* turns as well as the IR drop in the armature conductors.

The *energy lost* in the armature is

$$W_a = E_a \times I = (E - E_e - E_f)\, I. \tag{107}$$

E_e is the terminal or line E.M.F.

The *watts lost* in the *shunt field* are found by

$$W_s = E_s I_s = E_e I_s = R_s I_s^2. \tag{108}$$

The *series field loss* is given by

$$W_f = E_f I_e = R_f I_e^2. \tag{109}$$

The *eddy current** loss in the armature, from Steinmetz's formula, is

$$W_e = V(tfb)^2 \times 10^{-16}. \tag{110}$$

See Sec. 42 for (110). Several conditions may render the results of the actual tests on the dynamo widely diverse from these given in (110). Yet the latter will serve very well as a check.

* See Equation (58).

Hysteresis losses may be approximated by

$$W_h = VhfB^{1.6}. \qquad (111)$$

where the hysteretic constant h is known for the quality of iron used in the armature. Taking the following inductions in the armature, the hysteresis losses corresponding for soft iron or soft steel will be about as tabulated.

8. TABLE OF HYSTERESIS LOSSES.

B.	LOSS IN WATTS.
10,000	$W_h = Vf \times 6.28 \times 10^{-4}$
12,000	$W_h = Vf \times 8.40 \times 10^{-4}$
14,000	$W_h = Vf \times 1.08 \times 10^{-3}$
15,000	$W_h = Vf \times 1.20 \times 10^{-3}$

V is the volume of iron in cubic centimeters and f is the number of cycles per second.

The electrical efficiency is

$$\text{Elec. eff.} = \frac{E_e \times I_e}{E \times I}. \qquad (112)$$

The *commercial efficiency*, while it must be finally settled by experimental test, may be approximated by adding to the available output of the machine all the losses as just given, then divide the output by the sum. Hence

$$\text{Com. eff.} = \frac{\text{Output}}{\text{Output and losses}}. \qquad (113)$$

It is seen that no account is here taken of windage, bearing and brush friction, all of which reduce the efficiency, and which can be determined by actual test.

*58. **Illustrative Examples.** — 1. Let it be required to design a half horse-power shunt-wound motor for 110 volt circuits.

* These examples should also be worked out from gilberts per centimeter (H) taken from curves on p. 229, or from ampere-turns per centimeter taken from tables on p. 298.

As small a machine as this, especially if it runs light part of the time, cannot be expected to supply its power at greater than 60% to 70% efficiency. Hence the intake will be, say $\frac{746}{2} \div 0.60 = 625$ watts. At 110 volts there will be 5.6 amperes of current. Allow 400 circular mils per ampere, requiring $\frac{5.6}{2} \times 400 = 1120$ circular mils. This is nearest No. 19. We may approximate 10% I^2R losses in fields and in the armature, making each 62.5 watts lost. Suppose we use the allowable temperature rise as a basis for solving the armature in this case. If we allow 1 watt loss per square inch of external surface, we obtain the size as follows: let $d =$ the core diameter, $l =$ its length. Consider that these shall be equal. The end surfaces are $2\,d^2 \times 0.7854$. The convex surface is $d \times \pi \times l$. Since $d = l$,

$$2\,l^2 \times 0.7854 + \pi l^2 = 62.5,$$

and
$$l^2 = \frac{62.5}{4.7}. \quad \text{Whence } l = 4 \text{ inches.}$$

For the fields the current will be

$$10\% \text{ of } 5.6 = 0.56 \text{ ampere.}$$

This at 800 circular mils per ampere requires 448 circular mils, or No. 24 wire, having 26 ohms per 1000 feet cold. Assuming a rise in temperature of 40° C above the air, and taking 1% increase for each $2\frac{1}{2}°$ rise, the resistance, hot, will become about 116% of $26 = 30$ ohms per 1000 feet. The field coils must have a resistance of $110 \div 0.56 = 196$ ohms. Therefore the length $= 196 \div 30 = 6533$ feet.

From (79) $E = \frac{\phi n v}{10^8}$. Since $E = 110$, taking $v = 30$ r.p.s., assuming $B = 6000$ gausses, and 70% available armature cross-section, we have

$$n = \frac{110 \times 10^8}{(4 \times 2.54)^2 \times 0.70 \times 30 \times 6000} = 900 \text{ wires.}$$

Take, say 20 commutator segments, thus requiring $900 \div 20 = 45$ wires per coil and per slot, and 22 turns per coil. Divide the circumference of the armature into 40 parts, half of it being for slots. Circumference $= 4\pi = 12.56$ inches. Hence each slot will be $12.56 \div 40 = 0.3$ inch wide. No. 19 wire is 35.4 mils bare, or about 50 mils d.c.c. This will allow $0.31 \div 0.05 = 6$ wires in one layer in each slot; and the slots must then be $44 \div 6 = 8$ layers deep. The wire space in depth $= 0.05 \times 8 = 0.4$ inch. The slots must be insulated on the sides and bottom, say to the thickness of one wire. Hence 5 wires wide will require 9 wires deep + space for insulation = 10 wires deep. Depth therefore $= 0.05 \times 10 = 0.5$ inch. This will perhaps be also sufficient to allow for bond-wire space.

The total length of wire on the armature will be about 750 feet, allowing for piling up on the ends and for commutator connections. No. 19 has 8.5 ohms per 1000 feet. Hence the I^2R loss will be $\overline{5.6}^2 \times 1.613 = 50$ watts, or a little less than 10% as assumed at the outset. We get 1.613 as the hot resistance of 750 feet ÷ by 4 for the maximum armature resistance.

We may take the laminæ for the core 0.04 inch thick. The volume of the core $= \overline{10}^2 \times 0.7854 \times 10 = 785.4$ cc.

Applying table 8 under (111), taking the constant at 5 and calculating the hysteresis loss, we have

$$W_h = Vf \times 5 \times 10^{-4} = 785.4 \times 30 \times 5 \times 10^{-4}$$
$$= 12 \text{ watts.}$$

Also for eddy currents from (110)

$$W_e = V(tfb)^2 \times 10^{-16} = 785.4(40 \times 30 \times 6000)^2 \times 10^{-16}$$
$$= 4 \text{ watts.}$$

The total armature loss is $50 + 12 + 4 = 66$.

But since only half the external surface is iron the flux density will be doubled in the teeth, so that both hysteresis and eddy currents will be greater in the teeth. If necessary the volume of the teeth may readily be determined, also the volume of the rest of the core, with their corresponding flux densities, and the iron losses for these parts determined separately. This would perhaps increase the above value to 70 watts, or more.

Calculation of Field. — Assume the coefficient of leakage to be $v = 1.5$, thus making the lines of force in the cores of the field magnets become

$$\phi_f = 70 \times 6000 \times 1.5 = 630{,}000 \text{ maxwells.}$$

We shall make the core of wrought iron, but for purposes of rigidity it will be better to make it larger than otherwise necessary. Hence take $B = 8000$ gausses. Whence

$$D \text{ of cores } = \sqrt{\frac{630{,}000}{8000} \div 0.7854} = 10 \text{ cm.} = 4 \text{ in.}$$

Now if the poles and core are all of wrought iron 4 inches will be ample; but if cast iron be used the area of cross section must be somewhat larger so as to reduce B_f to

6000 gausses, perhaps. Use single yoke core. Allow a clearance of 0.08 inch, or 0.16 inch both sides.

Now to approximate the length of the magnetic circuit: from middle of pole face through pole to a point on a level with under side of the armature, say 3 inches; thence to top of field-winding, say 1 inch; thence to center of core, that is, wire and half of core depth, say 4 inches; yoke, say 4 inches, core length, making total approximate field circuit 20 inches. Also make poles of such shape and dimensions as that their cross-section will be approximately equal to that of the armature $= 4 \times 4 = 16$ sq. inches $= 100$ sq. cms.

Now for the field core the reluctivity is

$$k = \frac{a}{1 - bB} = \frac{0.0004}{1 - 0.000057 \times 8000}$$

$$= 0.00073 \text{ oersted per cc.}$$

$$\mathcal{R}_c = k\frac{l}{a} = 0.00073 \times \frac{10.16}{78.54} = 0.000094 \text{ oersted.}$$

The rest of the field will be 16 inches = 40.64 cms. long and 16 sq. inches = 103.25 sq. cms. section.

Hence $$B_f = \frac{630{,}000}{103.25} = 6000 \text{ gausses.}$$

And $$k = 0.0006 \text{ oersted per cc.}$$

$$\mathcal{R}_f = 0.0006 \times \frac{40.64}{103.25} = 0.00023 \text{ oersted.}$$

For the air gap, assuming all the lines to pass through it, the reluctance will be

$$\mathcal{R}_g = \frac{0.4}{100} = 0.004 \text{ oersted.}$$

Hence, neglecting the reluctance of the armature, the total reluctance will be

$$\mathcal{R}_t = \mathcal{R}_g + \mathcal{R}_f + \mathcal{R}_c = 0.004324 \text{ oersted.}$$

The approximate ampere-turns are

$$nI = \frac{\phi\mathcal{R}}{0.4\pi} = \frac{630{,}000 \times .004324}{1.256} = 2170.$$

Therefore the number of turns will be

$$n = \frac{2170}{0.56} = 3875 \text{ turns of No. 24 wire.}$$

This will have a diameter of 35 mils d.c.c. The length of wire already found is 6533 feet. Allow 20 per cent of this for the regulating rheostat, leaving

$$80\% \text{ of } 6533 = 5226.4 \text{ feet for field wire.}$$

The mean length of one turn will be $5226.4 \div 3875 = 1.35$ ft. mean circumference of winding. The mean diameter will be $\frac{1.35 \times 12}{3.14} = 5.2$ inches, making the depth of wire $5.2 - 4 = 1.2$ inches. Allow 0.1 inch for core insulation, and we have for wire space, $1.2 - 0.1 = 1.1$ inches, requiring $1.1 \div 0.035 = 31$ layers deep. Therefore the cores must be long enough for $3875 \div 31 = 125$ turns in one layer. The length for wire space is then

$$125 \times 0.035 = 4.38 \text{ inches.}$$

Allow $\frac{1}{8}$ inch for fiber collars at each end of core; this makes the total wire space 3.75 inches, shortage being made up in greater depth, if the four-inch core be used.

To correct our data. The core reluctance will not be appreciably changed, since the corrected core length is only a little different from that assumed. The diameter of pole bore is $4 + 0.16 = 4.16$ inches, circumference $= 13.06$ inches. Choose the angle of span $= 120°$, thus making the length of curve of pole face $= \frac{120}{180} \times \frac{13.06}{2} =$ 4.3 inches. The separation of pole tips will then be $\frac{13.06 - 8.6}{2} = 2.23$ inches. Pole face area $= 4.3 \times 4 =$ 17.2 sq. inches. We may so shape the poles that the average area will be about 16 sq. inches as assumed, and so no correction for field area need be taken.

Now to obtain exact length of flux lines. From the center of pole face horizontally 2 inches; vertically down 3 inches to top of winding; 1.6 inch depth of winding; 2 inches to core center. The total $= 6.6$ inches to a horizontal line through the center of core. Thence say 2.5 inches inward to core end, making 11.1 inches on one side exclusive of core. But the curving of the flux paths in the field parts will reduce this to say 10 inches, or 20 inches on both sides. Our assumed length was 16 inches. Hence the reluctance of this portion will be $\frac{5}{4}$ of the assumed value. Therefore $\frac{5}{4}$ of $0.00023 =$ 0.00029 oersted. The total reluctance will now be 0.004388, and the ampere-turns 2200 instead of 2170. Hence an additional layer of wire on the core will be sufficient, especially since our calculations assume that the total core flux passes through the complete field circuit.

Now a little time on the drawing-board will suffice to

represent the machine to scale, with the proper forming of the outline so as to present a satisfactory appearance and to save any iron possible whose removal will not prejudice the operation of the machine. Show substantial arms cast on the poles to support the armature bearings, and design feet to bolt on slide base.

Summary: — Armature 4″ × 4″; speed 1800 r.p.m.
Current 5.6 amps.; $R_a = 1.61$ ohms.
Wire length 750 feet; size No. 19 d.c.c.
Coils 20 having 22 turns each.
Laminæ 100, 0.04 inch thick.
Shaft $\frac{7}{8}$″.
Fields, core 4″ long, 4″ diameter.
Area 13 sq. in.; wire space 3.75″ in.
Flux length 10 inches each limb.
Total flux path 24″.
Current 0.56 ampere; turns 3875.
Wire length 5300 feet, No. 24 wire.
Span of poles 120°; separation 2.3″.
Losses, armature I^2R = 50 watts.
" Iron = 20 "
" Total = 70 "
Field I^2R = 62 "
Total = 132 "

$$\text{Efficiency} = \frac{746}{2} \div \left(\frac{746}{2} + 132\right) = 74\%.$$

2. Required to design a 5 K.W. bi-polar direct current machine for 120 volts full load.

As a check suppose we determine the size of the armature in two ways. First taking the peripheral speed at say 3600 feet per minute; also 1800 revolutions per

minute as a good speed for this size of machine. The diameter of the armature must be $\frac{3600}{1800\pi} = \frac{2}{\pi}$ ft. $= 8$ inches. Take also a length of 8 inches.

Second, if we take $1\frac{1}{2}$ sq. inches per watt of I^2R loss which according to the table will be 4% for a 5 K.W. dynamo, the diameter of the armature will come out as follows: $d^2 \times \pi + 2d^2 \times 0.7854 = 1.5 \times 200.$

$d = 8$ inches, nearly.

We shall therefore work on the basis of an armature $8'' \times 8''$, running at 1800 r.p.m. = 3780 feet per minute, peripheral velocity.

The armature current is

$$I_a = \frac{5000}{120} = 41.7, \text{ say } 42 \text{ amperes.}$$

At 600 circular mils per ampere, the wire must be $\frac{42}{2} \times 600 = 12{,}600$ cir. mils, which is between Nos. 10 and 9. Take No. 10 = 10,381 cir. mils. = 500 circular mils per ampere.

If we choose 4 volts between adjacent commutator bars there will be

$$2 \times \left(\frac{120}{4}\right) = 60 \text{ commutator bars.}$$

This might be made 50. Also

$$n = \frac{E \times 10^8}{\phi v} = \frac{E \times 10^8}{BAv}.$$

Deducting 20% from armature cross section,

$$A = 64 \times 0.80 = 51.2 \text{ sq. in.} = 330.24 \text{ sq. cms.}$$

Hence at $B_a = 10{,}000$ gausses, $\phi_a = 3{,}302{,}400$ maxwells. So that

$$n = \frac{120 \times 10^8}{3{,}302{,}400 \times 30} = 120 \text{ conductors} = 60 \text{ coils.}$$

If we choose 60 commutator bars there will be 1 turn in each coil. If we make 60 slots on the armature the wires will be 2 deep in each. Let the slots occupy half the surface; then each will be $\frac{8 \times \pi}{120} = 209$ mils wide. Now No. 10 wire = 101.4 mils uncovered, or about 120 mils d.c.c. The sides and bottom of the slots must be fibered, say 44.5 mils each to make up to about 209 mils; i.e., $120 + 2 \times 44.5 = 209$. Two wires deep make 240 mils, which added to, say, 50 mils insulation in the bottom, make 290 mils. Further, the bond-wires, say No. 18 = 40 mils, and the insulation under them, say 20 mils, make an additional depth of 60 mils, thus requiring the slots to be 350 mils deep.

Now if we may assume that the flux lines enter the teeth only, their density in the teeth will thus become 20,000 gausses, since only half the surface is iron.

The I^2R_a loss may now be estimated as follows: mean turn $= (8 - 0.35) \times 2 + (8 \times 2) = 31.3$ inches. Allow 25 per cent for rounding the shaft and piling up and making commutator connections, making the total length of wire on the armature equal

$$\frac{31.3 \times 1.25 \times 60}{12} = 195.625 \text{ feet.}$$

No. 10 has a resistance of 1.023 ohms per 1000 feet. Hence our armature wire will have a resistance of $0.1956 \times 1.023 = 0.2$ ohm. The resistance of the armature is therefore $\frac{1}{4}$ of $0.2 = 0.05$ ohm cold. To allow 40° C. rise will increase this about 16%, making 0.06 ohm hot. Hence

$$I^2R_a = (42 + 4.5\% \text{ of } 42)^2 \times 0.06 = 116 \text{ watts.}$$

We allow the armature current to increase 4.5 per cent to supply the field current. The per cent loss will now be $116 \div 5000 = 2.5\%$, or somewhat less than the tabular value for machines of similar size.

To calculate the fields. Take two cores, wrought iron; use cast iron poles, and the cast iron bed-plate for yoke. The leakage coefficient we shall assume to be $v = 1.4$ to be on the safe side. $\phi_f = 3{,}302{,}000 \times 1.4 = 4{,}623{,}000$ maxwells. Allowing $B_c = 15{,}000$ gausses, the core diameter will be

$$D_c = \sqrt{\frac{4{,}623{,}000 \div 15{,}000}{0.7854}} = 19.8 \text{ cms.} = 7.7 \text{ inches.}$$

Consider 4,000,000 lines to pass through the poles and $B_p = 8000$; then the area of poles will be

$$A_p = 500 \text{ sq. cms.}$$

Since the length of the pole bore = 8 inches = 20.32 cms., the curve of pole face must be $500 \div 20.32 = 24.5$ cms. Make the clearance $\frac{1}{8}$ inch, thus requiring the circumference of polar bore to be $8.25 \times \pi = 25.9$ inches = 65.78 cms. Therefore the separation of pole tips = $\frac{65.78 - 24.5 \times 2}{2} = 8.39$ cms. = 3.3 inches. The angle of span is $\frac{49}{65.78} \times 180^\circ = 134^\circ$. We shall let this stand as satisfactory. However, it would be possible to increase polar density somewhat, thus reducing the angle of span, which in turn would tend to reduce leakage.

There will, of course, be ample cross section in the bed-plate; hence B_y will perhaps not exceed 5000 gausses, or

6000 at the most. At 2 amperes exciting current the field loss is

$$I^2R_f = 120 \times 2 = 240 \text{ watts.}$$

There will be 120 watts dissipated in each coil which, if we use the rule of 2 square inches per watt, will have 240 square inches external radiating surface.

As a first approximate solution take $\frac{7}{8}$ inch as depth of wire on field core. The length of the core will be $240 \div [(7.7 + 2 \times \frac{7}{8}) \times \pi] = 8$ inches $= 20.32$ cms. The average length of the pole will be about 5 inches $=$ 12.7 cms. The finished coils should stand about 4 inches apart; therefore the yoke length will be about 13 inches $= 33$ cms.

For cores,

$$k = \frac{0.0004}{1 - 0.000057 \times 15{,}000} = 0.00276;$$

$$\mathcal{R}_c = 0.00276 \times \frac{20.32 \times 2}{308} = 0.000364.$$

For poles,

$$k = \frac{0.0026}{1 - 0.000093 \times 8000} = 0.0102 \text{ oersted per cc.;}$$

$$\mathcal{R}_p = 0.0102 \times \frac{12.7 \times 2}{500} = 0.00052 \text{ oersted.}$$

For yoke,

$$k = \frac{0.0026}{1 - 0.000093 \times 6000} = 0.00588;$$

$$\mathcal{R}_y = 0.00588 \times \frac{33}{600} = 0.00032 \text{ oersted.}$$

Next to find the armature reluctance, first find that of the teeth at $B = 20{,}000$, then of the rest of the armature

body at $B = 10,000$. The number of teeth under the pole $= \frac{134}{180} \times 30 = 22$ teeth whose area $= 22 \times 0.209 \times 2.54 \times (8 \times 2.54) \times 0.80 = 190$ sq. cms. Take $\mu = 30$ from table 13 at $B = 20,000$; $k = \frac{1}{\mu} = 0.033$. $\mathcal{R}_t = 0.033 \times \frac{0.889 \times 2}{190} = 0.000308$. Area of rest of armature $=$ $(20.32 - 1.778) \times 20.32 \times 0.80 = 301.38$ sq. cm. Length of lines through this, say 20 cms., allowing for curving round the shaft.

$$B_a = 3,302,400 \div 301.38 = 10,000, \text{ approximately.}$$

$$k_a = \frac{0.0004}{1 - 0.000057 \times 10^4} = 0.00093;$$

$$\mathcal{R}_a = 0.00093 \times \frac{20}{301} = 0.00006 \text{ oersted.}$$

Diam. of polar bore $= 65.78 \div 3.14 = 20.95$ cms.

Length of air gap $= 20.95 - 20.32 = 0.63$ cm.

$$\text{Area air gap} = \frac{500 + 190}{2} = 345 \text{ square cms.}$$

$$\text{Hence } \mathcal{R}_g = \frac{0.63}{345} = 0.00182 \text{ oersted.}$$

Armature,

$$(nI)_a = \frac{3,302,400\ (0.000308 + 0.00006)}{1.256}$$
$$= 968;\ n_a = 484.$$

Cores,

$$(nI)_c = \frac{4,623,000 \times 0.000364}{1.256}$$
$$= 1340;\ n_c = 670.$$

Poles,

$$(nI)_p = \frac{4,000,000 \times 0.00052}{1.256} = 1656;\ n_p = 828.$$

Yoke,

$$(nI)_y = \frac{4,000,000 \times 0.00032}{1.256} = 1018; \; n_y = 509.$$

Air,

$$(nI)_g = \frac{3,302,400 \times 0.00182}{1.256} = 4785; \; n_g = 2393.$$

Total ampere-turns $= 9767$; $n_t = 4883$ turns. $I^2 R_f =$ 240 watts; $R_f = \frac{240}{2^2} = 60$ ohms. The core diameter = 7.7 inches; assuming depth of wire $\frac{7}{8}$ inch, mean length of one turn $= \frac{\pi(7.7 + 0.875)}{12} = 2.24$ feet.

The total length of wire $= 2.24 \times 4883 = 10,938$ feet. Hence R_f per 1000 feet $= 60 \div 10.938 = 5.5$ ohms, corresponding approximately to No. 17 = 2048 circular mils, making 1024 circular mils per ampere of exciting current. No. 17 wire = 45.3 mils bare = 55 mils d.c.c. Allowing $\frac{1}{4}$ inch for collars on the core, there will be in one layer $\frac{8 - 0.5}{0.055} = 136$. Hence 2442 turns on each core require 18 layers deep = 0.99 inch. This exceeds $\frac{7}{8}$ inch given for the depth of wire; but the core need not be lengthened, since only 18 layers deep are required, $(0.99 \div 0.055)$. In one layer there will then be $2442 \div 18 = 136$ turns $= 136 \times 0.055 = 7.5$ inches for winding space. Adding $\frac{1}{4}$ inch at each end for a collar, we have 8 inches for total core length. Now this will not change the core reluctance nor ampere-turns from their assumed values of 0.000364 and 670 respectively. The total turns are 4896, or 2448 on each core. This again will make a slight difference in the resistance, which will now be $4896 \times 2.24 = 10.97$ thousand feet having $5.18 \times 10.97 = 56.8$ ohms; assuming 40° C.

rise, thus increasing the resistance by 16%, we have 65.8 ohms for the resistance of the shunt fields. This, however, can be reduced 10% to 15% to allow for rheostat resistance, bringing field resistance to about 60 ohms.

This calculation will not give constant potential under changing loads. Hence the series compensating turns must be determined. These must compensate the *cross* turns, and *back* turns, the armature drop, and the series field drop. Now by reference to Sec. 55 it will be seen that the cross ampere-turns on the armature produce a magnetism at right angles to that of the field. It will be nearly correct to consider that the resultant of the shunt field ampere-turns and the armature cross turns will be the whole number required. In other words, the difference between this resultant and the shunt ampere-turns will nearly represent the additional series ampere-turns to compensate for cross magnetization.

Cross turns $= \frac{134}{180} \times 60 = 45.$

Cross amp.-turns $\frac{1}{2} I \times n_c = \frac{44}{2} \times 45 = 990.$

The armature current is here taken as 44 amperes to include the shunt field exciting current, 2 amperes.

Back amp.-turns $(60 - 45) \times 22 = 330.$

Cross. mag. comp. amp.-turns $= (\overline{9792}^2 + \overline{990}^2)^{\frac{1}{2}}$
$= 9792 + 50.$

Series turns $= \frac{50}{42} = 1\frac{1}{5}$ turns for cross effect.

Series turns $= \frac{330}{42} = 7\frac{6}{7}$ turns for back effect.

Armature drop $= 44 \times 0.06 = 2.64$ volts $= 2.2\%$ of normal E.M.F.

Hence 2.2% of shunt ampere-turns are required to compensate it; 2.2% of 9792 = 216 ampere-turns, requiring series turns = 216 ÷ 42 = $5\frac{1}{7}$ turns for armature drop.

Total series turns = $1\frac{1}{5} + 7\frac{6}{7} + 5\frac{1}{7} = 15$. The wire should be taken large enough to reduce the drop in the series field coils to a minimum. Assume 1000 circular mils per ampere. The wire must then have 42 × 1000 = 42,000 cir. mils, between No. 4 and No. 3. Say we choose No. 4 = 0.2 inch bare = about 0.25 inch d.c.c. The winding space then is 15 × 0.25 = 4 inches, or 2 inches on each core. Length of wire $= \frac{(7.7 + 1.98)\pi \times 15}{12} =$ 38 feet having a resistance of 0.0095 ohm.

$$\text{Series field drop} = 0.0095 \times 42 = 0.40 \text{ volt.}$$

This is so small that the excess turns taken for compounding effects, and the margin on the shunt field will amply compensate. However, an extra series turn may be added, then the rheostat set to produce proper running voltage.

Now if it is desired to overcompound the machine, say 5%, an additional number of series ampere-turns may be put on equal to 5% of the shunt field ampere-turns. This machine would require 5% of 9792 = 490 ampere-turns, and say 12 series turns.

If in designing, the induction allowed in the fields is kept well within reasonable limits, the added series ampere-turns will not affect the saturation of the iron sufficiently to lower appreciably its permeability.

Summary: Machine 120 volts, 42 amperes.
Armature, 8″ × 8″; 1800 r.p.m.; $B_a = 10{,}000$.

$I_a = 44$; wire No. 10, 60 coils, 60 slots.
Wire length 195.6 feet; $R_a = 0.06$.
Volts drop 2.6; $\phi_a = 3.3024 \times 10^6$.

Fields: Core diam. 7.7″; $B_c = 15{,}000$.
$\phi_c = 4.623 \times 10^6$; shunt turns 2448 each.
Exciting current 2 amperes.
Wire length 10,970 ft.; size No. 17.
$R_f = 60$ ohms.
Series turns 15; No. 4 wire; length 38 ft.

The efficiency, so far as we are able to calculate it, is found as follows: —

Hysteresis,

$$W_h = \overline{20.32}^2 \times 0.7854 \times 20.32 \times 6.28 \times 10^{-4} \times 30$$
$$= 125 \text{ watts.}$$

Eddy current,

$$W_e = 6589\,(40 \times 30 \times 10{,}000)^2 \times 10^{-16}$$
$$= 95 \text{ watts.}$$

$I^2R_a = 116$ watts.
Total armature loss = 336 watts.
Series field loss = $0.0095 \times \overline{42}^2 = 16.76$ watts.
Shunt field loss = 240 watts.
Total field loss = 256.76 watts.
Total losses = 592.76 watts.
Per cent loss = $592.76 \div (5000 + 596.76) = 10\%$.
Efficiency = $100 - 10 = 90\%$.

3. Determine the essential data for a 10-K.W., 250-volt, 4-pole, cylindrical ring armature, direct current machine. Series connect the armature conductors so that only two brushes are used.

By the methods already used, making proper allowances for internal ventilation, taking the I^2R_a loss equal to 3%, we obtain for the armature dimensions, length 10 inches, outside diameter 10 inches, inside diameter 6 inches, thus making the thickness of the ring 2 inches.

Take a moderate speed, say 1200 r.p.m.

Whence $$250 = \frac{\phi n v p}{10^8}$$

from which $$\phi n = \frac{250 \times 10^8}{20 \times 2} = 6.25 \times 10^8.$$

We may reasonably allow $n = 200$, thus giving

$$\phi = 3,125,000 \text{ maxwells.}$$

The circumference $= 10\pi = 31.416$ inches.

Take one-half this $= 15.708$ inches for slots. The machine current is 40 amperes, 20 amperes in each conductor, which at 500 circular mils per ampere requires the wire to be

$$20 \times 500 = 10,000 \text{ cir. mils} = \text{No. } 10.$$

This has an area of 7854 square mils. Take flat wire whose width = 3 times its thickness. For 7854 square mils the wire must therefore be 51 mils × 153 mils bare, and double cotton covered = 75 × 200 approximately.

The slots must therefore be

$$\text{Width} = 200 + 40 = 240 \text{ mils.}$$

Hence $$\text{No. slots} = 15.708 \div 0.240 = 65, \text{ say } 66.$$

Putting 3 wires in each slot makes 198 surface conductors. The speed must be slightly increased to compen-

sate for the two conductors dropped; otherwise carry the field magnetization slightly higher.

Depth of wire $= 3 \times 75 = 225$ mils; allow 100 mils for insulation in the bottom of the slots, thus requiring the depth to be 325 mils.

Exact width of each slot will be

$$31.416 \div (66 \times 2) = 0.238 \text{ inch.}$$

This is also the width of each tooth.

The approximate length of wire will be

$$\frac{66\,[(10 + 10 + 2 + 2) + 6]}{12} = 165, \text{ say } 175 \text{ feet.}$$

No. 10 has 1.023 ohms per 1000 feet; hence

$$R_a = \frac{(1.023 \times 0.175) + 16\% \text{ of } (1.023 \times 0.175)}{4}$$

$$= 0.0525 \text{ ohm.}$$

Allowing 0.1 inch clearance, the pole circle will have a diameter

$$D = 10 + 0.2 = 10.2 \text{ inches, and}$$

$$\text{Circum.} = 10.2\pi = 32 \text{ inches.}$$

Making the width of the poles equal to their distance apart, we have

Pole width $32 \div 8 = 4$ inches.

Pole face length $= 10$ inches.

Area pole face $= 10 \times 4 = 40$ sq. in. $= 250$ sq. cms.

Pole flux, perhaps, $3.125 \times 10^6 \times 1.25 = 4 \times 10^6$ maxwells.

$$B_p = 4 \times 10^6 \div 250 = 16{,}000 \text{ gausses.}$$

This intensity may be reduced by making the width of the pole face slightly greater. To determine approximately the field winding. Take the length of poles = 6 inches = 15.24 cms. Also

$$\text{Thickness of pole ring} = \frac{\frac{1}{2}(4 \times 10^6) \div 12{,}000}{10 \times 2.54} = 7 \text{ cms.}$$

$$= 2.5 \text{ inches.}$$

Mean circum. of ring $(5.1 + 6 + 1\frac{1}{4}) \times 2\pi \times 2.54 = 197$ cms.

Length of flux travel in pole ring is

$$l = \tfrac{1}{4} \text{ of } 197 = 49 \text{ cms.}$$

For curving down to enter poles add $3 + 3 = 6$ cms. making 55 centimeters.

Armature, Norway iron, $l = 10'' = 25.4$ cms., for lines.

Area, $A = (10 \times 2.54) \times (2 \times 2.54) = 129$ sq. cms.

$$B_a = \frac{3.125 \times 10^6}{2} \div 129 = 12{,}100 \text{ gausses.}$$

$$k = \frac{0.0001}{1 - 0.000059 \times 12{,}100} = 0.00035 \text{ oersted per cc.}$$

$$\mathcal{R}_a = 0.00035 \times \frac{25.4}{129} = 0.00006 \text{ oersted.}$$

Air gaps, length, $l = 0.2$ inch $= 0.508$ cm.

$$\text{Area, } A = \frac{250}{2} = 125 \text{ sq. cms.}$$

$$\mathcal{R}_g = \frac{0.508}{125} = 0.004 \text{ oersted.}$$

Cores, wrought iron, $l = 15.25$ cms.

$$A = \frac{250}{2} = 125 \text{ sq. cms.}$$

$$B_c = \frac{4 \times 10^6}{2 \times 125} = 16{,}000 \text{ gausses.}$$

$$k = \frac{0.0004}{1 - 0.000057 \times 16{,}000} = 0.0045 \text{ oersted per cc.}$$

$$\mathcal{R}_c = 0.0045 \times \frac{15.25 \times 2}{125} = 0.0011 \text{ oersted.}$$

Yoke, $l = 55$ cms.

$A = 175$ sq. cms.

$$B_y = \frac{4 \times 10^4}{2} \div 175 = 11{,}400 \text{ gausses.}$$

$$k = \frac{0.0004}{1 - 0.000057 \times 11{,}400} = 0.00114 \text{ oersted per cc.}$$

$$\mathcal{R}_y = 0.00144 \times \frac{55}{175} = 0.00045 \text{ oersted.}$$

Reluctance of a single circuit is

$$\mathcal{R} = 0.00006 + 0.004 + 0.0011 + 0.00045$$
$$= 0.0056 \text{ oersted.}$$

Of two circuits in parallel the reluctance is

$\mathcal{R} = 0.0028$ oersted.

M.M.F. $= 0.0028 \times 3.125 \times 10^6 = 8750$ gilberts.

Ampere-turns, each core, $= \dfrac{8750 \div 2}{1.256} = 3483.$

Exciting current $\dot{I}_s = 3.5\%$ of $40 = 1.5$ amperes.

Hence

$n = 2322$, say 2325 turns, each core.

Total turns, $n = 2325 \times 4 = 9300.$

$$R_f = \frac{250}{1.5} = 166.6 \text{ ohms} = 41.6 \text{ ohms each core.}$$

Assume for calculation that the depth of winding is 1 inch. Hence the mean turn = 32″ = $2\frac{2}{3}$ feet.

Total length of wire each core = 6200 feet.

The 41.6 ohms must be the hot resistance of the field, making about 37 ohms cold, which corresponds to No. 18 wire = 40 mils bare = 55 mils d.c.c.

Assume core length = 6 inches exclusive of collars and insulation. This will permit in one layer

$$6 \div 0.055 = 109 \text{ turns, and}$$

$$\begin{aligned}\text{Number of layers deep} &= 2325 \div 109 \\ &= 21, \text{ and } 36 \text{ turns over.}\end{aligned}$$

$$\text{Depth of winding} = 0.055 \times 22 = 1.2 \text{ inches.}$$

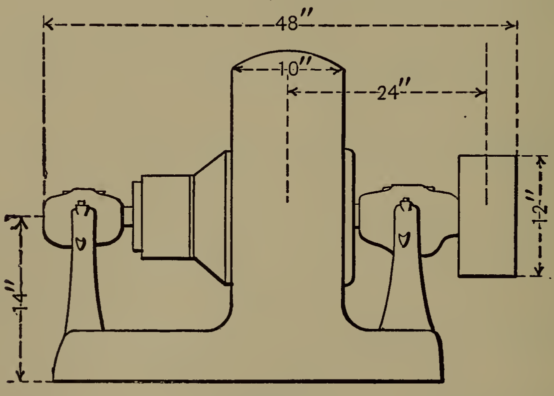


Fig. 20.

Hence it will not be necessary to modify our assumed dimensions except for symmetry in our drawing. Figs. 20 and 21 show the general dimensions of the finished machine.

Volts drop in the armature = 0.0525 × 40 = 2.1.

Hence the series ampere-turns to compensate will be

$$\frac{2.1}{250} \times 2325 = 20 \text{ on each core.}$$

$$\text{Number turns} = \frac{20}{40} = \tfrac{1}{2} \text{ to each core.}$$

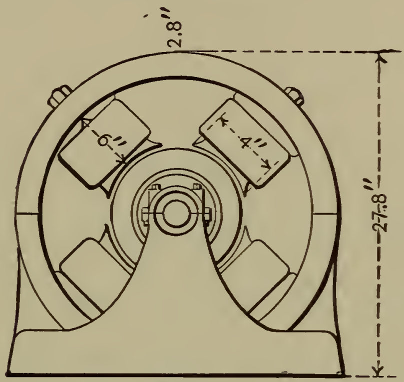


Fig. 21.

To compensate cross and back turns will require an additional 5% excitation.

$$5\% \text{ of } 2325 = 117 \text{ ampere-turns.}$$

Hence $$n = \frac{117}{40} = 3 \text{ turns each core.}$$

This makes a total of about 4 series turns on each core.

Length approximately = 3 × 16 = 50 feet, say of No. 2, thus giving 800 cir. mils per ampere.

$$R = 0.050 \times 0.16 = 0.008 \text{ ohm cold.}$$

$$\text{Drop} = 0.008 \times 40 = 0.32 \text{ volt.}$$

The excess of series turns already put on will compensate for this drop.

Calculate the efficiency of this machine and summarize the results of the problem. Calculate the overcompounding for 5% line loss.

4. Determine the data for a $\frac{1}{4}$ H.P. bipolar motor for 112 volts direct current.

In this design we shall express our dimensions in inches, and make use of table 13 and the curves in table 9 in order to illustrate their application.

We have seen that reluctance

$$\mathcal{R} = k\frac{l}{a} = \frac{l}{a\mu} \cdot \quad \text{Also,} \qquad (114)$$

$$nI = \frac{M}{1.256} = \frac{\phi\mathcal{R}}{1.256} = \frac{\phi \times l}{a\mu \times 1.256} \cdot \qquad (115)$$

Here l is expressed in centimeters and a in square centimeters. Now, if these dimensions are expressed in inches and square inches, the numerator of the fraction must be multiplied by 2.54, and the denominator multiplied by 6.45. Introducing these values in the equation it becomes

$$nI = \frac{\phi \times l \times 2.54}{1.256 \times a \times \mu \times 6.45} = \frac{0.3132 \times l \times \phi}{a\mu} \cdot \qquad (116)$$

This gives the ampere-turns for any one portion of the magnetic circuit. So that knowing l and a we have only to determine μ from the flux density by reference to tables 13 and 9.

For $\frac{1}{4}$ H.P. at 112 volts the current required will be about 2 amperes, or one ampere in each armature conductor. For so small a machine we shall allow ample cross section in the armature conductors and select No. 21, having 810 circular mils, diameter 0.034 in., double cotton covered. Calculations as before, also reasonable assumptions for an armature of this power will give a diameter of about 3 inches. Take a length of 3 inches. Then for a speed of, say 2400 r.p.m., and say 16 slots

holding 6 wires in width and 9 deep, or 54 wires each, and $54 \times 16 = 864$ conductors total, the required armature flux will be

$$\phi = \frac{10^8 \times 112}{40 \times 864} = 324{,}074 \text{ maxwells.}$$

This will make B about 60,000 per square inch. Making proper allowances for the insulation, the slots must be $\frac{3}{8}$ inch wide by $\frac{1}{4}$ inch deep. Therefore the cross sectional area of the body of armature iron is

$$3 \times [3 - (\tfrac{3}{4} + \tfrac{5}{8})] = 4.875 \text{ square inches,}$$

since the shaft will be $\frac{5}{8}$ inch and the depth of two teeth is $\frac{3}{4}$ inch. This will make the density

$$324{,}074 \div 4.875 = 66{,}476.$$

The length of the flux lines in the armature body will be about 1.75 inches.

The width of teeth at the top = 0.3125 inch; at the bottom, 0.1875 inch; average, 0.25 inch.

$$\text{Area} = 0.25 \times 3 \times 6 = 4.5 \text{ sq. in.}$$

The density in the teeth $= 324{,}074 \div 4.5 = 72{,}016$ lines per square inch. The average width of the tuft of lines entering each tooth, experience teaches, will be the width of the top of tooth plus the length of the air gap. Hence with 6 teeth under the pole at a time the air gap area will be

$$(0.3125 + 0.062) \times 3 \times 6 = 6.741 \text{ sq. in.}$$

The field flux will be, say $324{,}074 \times 1.1 = 356{,}481$. We shall make the field rectangular in shape with inwardly projecting horizontal poles. Make the poles square in cross section with slightly rounded corners, say 9 square inches area. The pole density will be

$356{,}481 \div 9 = 39{,}609$. Allow the wire to wind deeper in this small machine and shorten the poles so that about $1\frac{1}{2}$ inches will be its depth. This will require about $1\frac{1}{2}$ inches length of winding space. Adding $\frac{1}{8}$ inch each for collars makes $1\frac{3}{4}$ inches for length of poles, say $1\frac{7}{16}$ inches to allow for portion outside of coil.

From the poles the lines of force will have two parallel paths, and so the cross section of the yoke frame need be only one-half that of the poles. It will be 3 inches wide, of course, requiring a thickness of yoke of $1\frac{1}{2}$ inches. We shall now have between ends of field coils $3 + \frac{1}{16} + \frac{1}{16} + \frac{1}{8} = 3\frac{1}{4}$ inches. From center of yoke on one side through field coil, air gaps, field coil to center of yoke on the opposite side $1\frac{1}{2} + 1\frac{3}{4} + 3\frac{1}{4} + 1\frac{3}{4} = 8\frac{1}{4}$ inches. The height from center of horizontal portion of yoke (base) at the bottom to center of horizontal yoke at the top about $7\frac{3}{4}$ inches. The over-all length of the rectangle through the poles and armature will be $8\frac{1}{4} + 1\frac{1}{2} = 9\frac{3}{4}$ inches. Over all height $= 7\frac{3}{4} + 1\frac{1}{2} = 9\frac{1}{4}$ inches.

We may now summarize and calculate the ampere-turns required for cast iron fields and wrought iron armature.

Armature: $l = 1.75$ inches; $A = 4.875$ square inches.
μ, from table 13 for density $= 66{,}475$ $= 10{,}300$ per sq. cm., is about 2000;
also from 9, $\mu = \dfrac{B}{H} = \dfrac{10{,}300}{5}$ $= 2000$.

$$(nI)_a = \frac{0.3132 \times l \times \phi}{a \times \mu}$$

$$= \frac{0.3132 \times 1.75 \times 324{,}074}{4.875 \times 2000} = 18.$$

Also using the column giving ampere-turns per cm. length at $B = 10{,}300$, we obtain $\frac{nI}{l} = 4$, approximately.

Hence $(nI)_a = (1.75 \times 2.54) \times 4 = 17.6$, or 18 approximately.

Teeth: $l = \frac{3}{8}$ inch; A (under pole) $= 4.5$ sq. in.

μ, for a density of $\frac{72{,}016}{6.45} = 11{,}200$ gausses, is 1752.

$$(nI)_a = \frac{0.3132 \times \frac{3}{8} \times 324{,}074}{4.5 \times 1752} = 4.8.$$

Also for $B = 11{,}200$, $\frac{nI}{l} = 4.9$, and $(nI)_t = 4.9 \times (\frac{3}{8} \times 2.54) = 4.6$.

Air gap:

$l = \frac{1}{16}$ inch; $A = 6.741$ sq. in.; $\mu = 1$.

$$(nI)_g = \frac{0.3132 \times 324{,}074 \times 1}{6.741 \times 1 \times 16} = 942.$$

The density in air gap $= 7454$ gausses $= H$ gilberts per cm. length. This is $(650 \times 10) + (20 \times 44)$, approximately, making

$$\frac{nI}{l} = (520 \times 10) + (35 \times 20) = 5900,$$

from table 13.

$$nI_g = 5900 \times (\tfrac{1}{16} \times 2.54) = 944.$$

Poles: $l = 2$ inches average; $A = 9$ sq. in.

μ, for a density $= \frac{39{,}609}{6.45} = 6140$, is 250, approximately.

Also from cast iron curve in 9 at $B = 6140$, $H = 24$, and

$$\mu = \frac{B}{H} = \frac{6140}{24} = 250 \text{ as before.}$$

$$(nI)_p = \frac{0.3132 \times 2 \times 356{,}481}{9 \times 250} = 97.5, \text{ approximately.}$$

Also from table 13, $(nI)_p = 18.5 \times (2 \times 2.54) = 96$.

Yoke: $l = 8.25$ inches from base of pole half way around each way; $A = 9$ sq. in. for two parts in parallel.

$\mu = 250$, as before, for density of 6140 gausses.

$$(nI)_y = \frac{0.3132 \times 8.25 \times 356{,}481}{9 \times 250} = 402.$$

Also from 13, $(nI)_y = 18.5 \times (8.25 \times 2.54) = 388$.

Total ampere-turns in one-half of magnetic circuit =
$9 + 4.8 + 942 + 97.5 + 402 = 1455.3$, say 1456.

For both halves there are required $1456 \times 2 = 2912$.

It will be noticed by use of the column giving the ampere-turns per cm. length, and multiplying by length of part, we get in general a different value from that just obtained. This is partly due to the approximation which has to be made to the value of $\frac{nI}{l}$, where it falls between the tabular values, and to differences in the iron forming the bases of the values in 13 and 9.

Each pole will be wound for 1456 ampere-turns. For a depth of $1\frac{1}{2}$ inches the mean length of 1 turn = 1.169 ft. We may then use for the size of the wire the formula,

$$d^2 = \frac{k \times \text{amp.-turns} \times \text{mean length of 1 turn}}{\text{E.M.F.}} \qquad (117)$$

$$= \frac{12 \times 2912 \times 1.169}{112} = 350 \text{ cir. mils.}$$

k is taken at 12 instead of 10.79, to allow for a temperature rise of about 35° to 40° C.

This size is between No. 25 and 24. Use No. 24. In one pound of No. 24 there are 724.64 feet of double cotton covered wire. Hence one pound will wind

$$\frac{724.64}{1.169} = 620 \text{ turns.}$$

No. 24 wire has a resistance of about 29 ohms per 1000 feet, giving for 1 pound

$$\frac{724.64}{1000} \times 29 = 21 \text{ ohms.}$$

For 1 pound at 112 volts,

$$I = \frac{112}{21} = 5.33 \text{ amperes.}$$

Ampere-turns for 1 pound $= 620 \times 5.33 = 3306$, which is the same number that would be obtained with any number of pounds of the same size of wire, neglecting heating effect; for as the weight of wire would increase the resistance would increase and the current reduce proportionately; but the number of turns would be increased in the same proportion; hence the ampere-turns would remain the same.

To reduce current to a permissible amount for this size, as well as for economy, we may take 10 pounds, requiring $5.33 \div 10 = 0.533$ amperes, and giving $620 \times 10 = 6200$ turns, and

$$6200 \times 0.533 = 3306 \text{ ampere-turns.}$$

This is slightly larger than calculated, but it will give some margin for regulation by rheostat. Furthermore the rise

of temperature will somewhat reduce the field current. Altogether this will make a very satisfactory winding.

Calculate the depth of winding for the above number of turns. It will be approximately $1\frac{3}{8}$ inches.

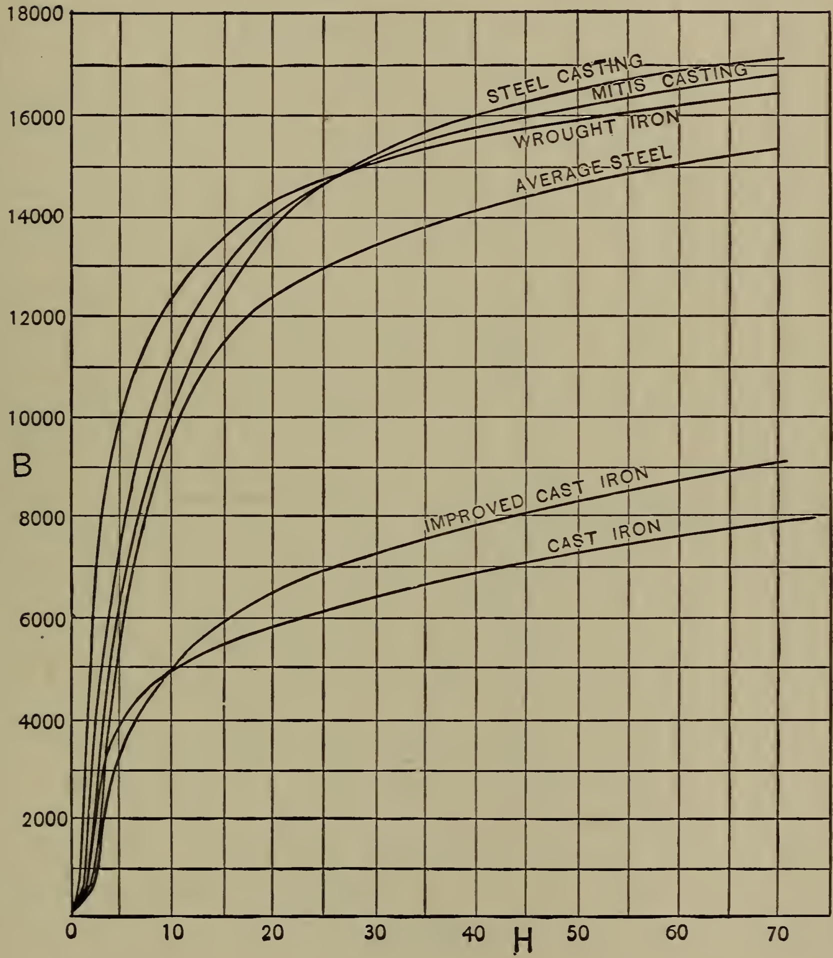
Calculate the length and weight of wire for winding the armature, as previously indicated.

The pedestals for support of shaft will be $\frac{3}{4}$ inch wide next to shaft, and 2 inches at the foot, with a projection for bolting to base. They will be $\frac{5}{8}$ inch thick and cut out to a web about $\frac{1}{8}$ inch thick. The arms on the base for support of pedestals will be, for the longer one on the commutator end, $3\frac{5}{8}$ inches long to center of pedestal, by $2\frac{1}{4}$ inches wide; for the shorter one $1\frac{7}{8}$ inch to the center of pedestal, by $2\frac{1}{4}$ inches wide. They will be cut out underneath, leaving only $\frac{1}{4}$ inch of thickness of iron all around, and lag-bolted against the lower portion of yoke. The pedestals will be, from center of shaft to base, $3\frac{1}{8}$ inches. The shaft is $\frac{5}{8}$ inch through the armature core and turned to $\frac{3}{8}$ inch outside. It will be 9 inches long.

Length over all = $9\frac{3}{4}$ inches.
Width over all = $9\frac{1}{2}$ inches.
Height over all = $9\frac{1}{4}$ inches.

Make a complete drawing of a motor of this design, including two elevations and plan of base and showing all necessary dimensions for its construction. Summarize all the work, giving all dimensions, sizes, weights, etc. Where necessary design exact details. For example, draw the details for the brush-holder, rocker arm, sixteen part commutator and pedestal.

9. B-H CURVES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STEEL CASTING
MITIS CASTING
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XII.

ALTERNATING CURRENTS.

59. **General Definitions.** — The nature of alternating currents necessitates the use of certain quantities which do not occur at all in direct current phenomena. Hence it will be well at the outset to gain some familiarity with these terms.

(*a*) If the magnetic field in which a closed coil of wire revolves be uniform, the E.M.F. generated in the coil will vary as the sine of the angle through which it is turned, the rate being uniform; in other words the curve of variation will be a *sine curve;* the heights of the curve, or its *ordinates*, representing the E.M.F's. at the succes-

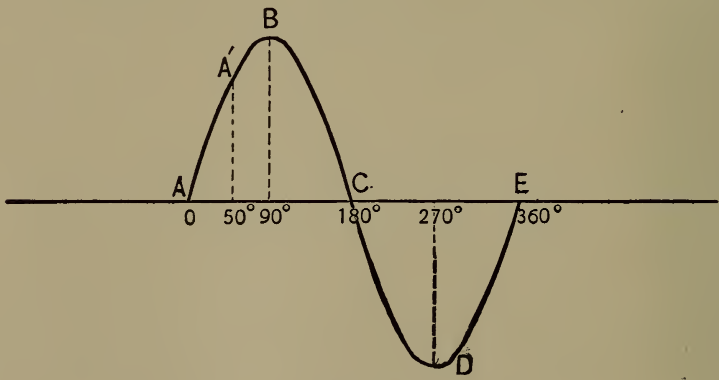

Fig. 22.

sive points will be proportional to the sines of the corresponding angles through which the coil has turned from the position of zero E.M.F. The E.M.F. is zero when

the plane of the loop is perpendicular to the lines of magnetic force. Fig. 22 shows the sine curve. Distances measured horizontally represent angles, distances vertically represent E.M.F's.

The height of the curve A' above the horizontal line represents the E.M.F. in the coil when revolved 50° from the position when the plane of the coil is normal to the lines of force. The height at B is proportional to the E.M.F. at 90°, or in the position when the coil's plane is parallel with the lines of magnetic force. This is clearly the position of maximum positive E.M.F. Therefore

$$\frac{\text{E.M.F. at } A'}{\text{E.M.F. at } B} = \frac{\sin 50^\circ}{\sin 90^\circ}.$$

Hence the name *sine curve.*

(*b*) It is clear from the curve that the *average* E.M.F. during one-half a revolution, or one alternation, is proportional to the average ordinate of the curve. Call the maximum ordinate at 90° one, then the area of the curve between the horizontal line and ABC is 2. The length $AC = 180^\circ = \pi = 3.1416$. Therefore the average ordinate is

$$\frac{\text{area}}{\text{length}} = \frac{2}{\pi} = 0.637 \text{ of the maximum.}$$

Therefore, calling the maximum E.M.F. E, and the average E.M.F. E_a, we have

$$E_a = 0.637E. \qquad (118)$$

Suppose the maximum pressure be 100 volts, then the average pressure will be.

$$E_a = 0.637 \times 100 = 63.7 \text{ volts.}$$

(*c*) The *effective pressure* of alternators is some greater than the *average*, and is that which is available for producing heat if applied to a resistance. The *effective pressure* is the square root of the mean of the squares of the successive pressures during one-half a revolution of the pressure coil, because the successive heats are proportional to the successive E^2's. The E.M.F's. vary from zero to 1 as a maximum. Therefore the mean of the squares is, by adding the squares of say 12 ordinates, and dividing by 12, $\frac{1}{2}$.

Its square root is $\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = 0.707$ of the maximum.

Calling the effective E.M.F. E_e, we have

$$E_e = 0.707\ E. \tag{119}$$

For a maximum pressure of 100 volts the effective E.M.F. will be (that indicated by a voltmeter)

$$E_e = 0.707 \times 100 = 70.7 \text{ volts.}$$

(*d*) The *period* of an alternating current is the time from A in Fig. 22 to E, or the complete cycle, or one complete revolution of the coil in a 2-pole machine. The number of complete periods per second is the *frequency* of the current. A cycle consists of two *alternations*, or reversals of current. Therefore a frequency of 125 per second means $125 \times 2 = 250$ alternations per second. The frequency of practical machines in the U. S. will nearly always be 60 or 25 per second, although 125, 40, 66, 100 and 133 are used. A 4-pole machine gives a frequency equal to twice the number of revolutions per second; an 8-pole, 4 times the number.

Hence to obtain the frequency f of any machine, multiply the number of revolutions per second by the number of pairs of poles p. An 8-pole machine running at 2400 revolutions per minute has a frequency

$$f = \frac{2400}{60} \times 4 = 160.$$

(*e*) An *electromotive force of self-induction* is set up in a circuit counter to the changing current caused to flow in the circuit. This counter E.M.F. is the result of the varying lines of force, set up by the variable current, which thread the adjacent portions or coils of the circuit. If this E.M.F. is set up by causing the lines of force to pass through the loops of an adjacent coil, the electromotive force is that of *mutual induction.* The direction

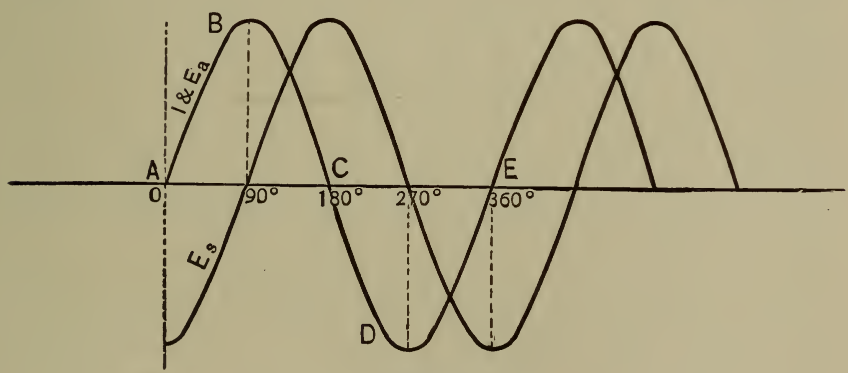

Fig. 23.

of the induced E.M.F., whether of self-induction or of mutual induction, is, according to Lenz's law, in the direction to oppose the inducing force. This must also follow from the law of the conservation of energy; otherwise any empty circuit, say the secondary of a transformer, would absorb as much energy as a loaded one.

Since E.M.F. is proportional to the *rate of change* of the number of lines of force threading any circuit or loop, the self-induced pressure is the greatest when the alternating current is the least, or zero, and vice versa. Although at its maximum value the current sets up the maximum number of lines, the *rate* of *change* of the lines is the least; while at zero current and lines the rate of change is a maximum, hence the induced E.M.F. is a maximum. A reference to Fig. 23 will make clear the relative values of the inducing current and E.M.F. of self-induction at successive instants. It will be observed that the E.M.F. of self-induction E_s lags 90° behind the curve of current and active E.M.F., E_a. For an instant at B and D there is no change in the current, hence no induced E.M.F., while the change is most rapid through C and E shown by the slope of the curve, and hence there is a maximum counter E.M.F. at these points.

(*f*) Whenever an alternating pressure is applied to a circuit a certain component of it is useful to send the current through the resistance. This component is called the *active pressure.* The other component at right angles to the first goes to balance the E.M.F. of self-induction set up by the alternating current; this is sometimes called simply the *self-induction pressure.* The total applied E.M.F is called simply the *impressed pressure.* This is analogous to a vessel moving, say N. of E. at a given *impressed* speed. It will be going east at a certain speed which we may call the *active* speed. We may suppose, in fact, the desired course is E., but on account of adverse currents or winds, the counter pressure, it has to steer N. of E. It will at the same time be moving north at a cer-

tain speed, which we may call briefly the *counter* or *self-induction* speed. These two are at right angles, or 90° apart in *phase.*

As already shown, the counter E.M.F. lags 90° behind the current; therefore it is 90° back of the *active pressure*, that is, the phase difference is 90°, as shown in Fig. 23.

The E.M.F. of self-induction being proportional to the rate of change of the current and active pressure is proportional to the angular velocity or displacement of the coil in which the induction takes place, or to $2\pi f$, f being the frequency; it is also proportional to the *coefficient* of *self-induction*, a quantity depending upon the position, winding, etc., of the coil itself. Hence we can express the counter E.M.F. by writing

$$E_s = 2\pi f L I. \tag{120}$$

The resistance due to self-induction is

$$R_s = 2\pi f L. \tag{121}$$

The active pressure is written

$$E_a = IR. \tag{122}$$

in which R is the ohmic resistance of the circuit.

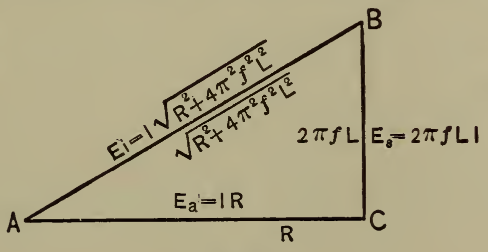

Fig. 24.

The relation of the different quantities in an inductive circuit may be expressed by a diagram, Fig. 24. The

counter E.M.F. is represented by the vertical CB, and the active by AC, the impressed E_i being the hypotenuse of the triangle of which the two components E_a and E_s are the two perpendicular sides.

The angle BAC is the *angle of lag* of the current and active pressure behind the impressed pressure AB, which is

$$E_i = I\sqrt{R^2 + 4\pi^2 f^2 L^2}. \tag{123}$$

This is Ohm's law applied to alternating currents. Transposing this so as to put it in the usual form, we have for current in alternating current circuits,

$$I = \frac{E}{R} \frac{1}{\sqrt{1 + \frac{4\pi^2 f^2 L^2}{R^2}}}. \tag{124}$$

If $L = 0$, then $I = \frac{E}{R}$ as in direct current circuits. $2\pi f$ is often represented by ω, the angular velocity; in which case (124) would be written

$$I = \frac{E}{R} \frac{1}{\sqrt{1 + \frac{\omega^2 L^2}{R^2}}}. \tag{125}$$

(g) If the values of the various E.M.F's. in Fig. 24 be divided by I, the current, the quotients will be the equivalent resistances as shown inside the diagram. R is the metallic or *ohmic* resistance; $2\pi f L$ is the *inductive* resistance, or reactance; $\sqrt{4\pi^2 f^2 L^2 + R^2}$ is the apparent resistance or *impedance.* The hypotenuse, of course, is the square root of the sum of the squares of the two sides.

(h) In the above equations L is the coefficient of *self-induction*, a constant for any given circuit without iron. It is defined as the *E.M.F. induced in a circuit wholly free*

from iron or other magnetic material, when the current varies at the rate of one ampere per second. It is also called the *inductance* of the circuit. The unit of inductance is the henry, or that of 1 volt of counter E.M.F. when the current changes at the rate of 1 ampere per second. The henry $= 10^9$ C.G.S. units of inductance.

(*i*) The *capacity* of an alternating current circuit is the measure of the amount of electricity held by it when its terminals are at unit difference of potential. Every such circuit acts as a condenser, and a current will flow back and forth, even though it is open and in the ordinary sense unloaded, proportional to the rate of change of the active pressure of the circuit, and to the capacity of the circuit. The effect of capacity is directly opposite to self-induction. Hence by properly arranging the capacity of a circuit it is possible to neutralize inductance, and so to bring the alternating current under the same laws as direct. If f be the frequency and E_c the pressure at any moment applied to a condenser, or circuit whose condenser capacity is C, the current flowing back and forth due to this capacity is

$$I = 2\,\pi f C E_c. \tag{126}$$

and

$$E_c = \frac{I}{2\,\pi f C}. \tag{127}$$

In this case the resistance due to capacity is

$$R_c = \frac{1}{2\,\pi f C}. \tag{128}$$

E_c may be called the *capacity pressure* which is opposite to

E_s, the self-induction pressure. Fig. 25 represents the relation of capacity to other elements of an alternating current circuit.

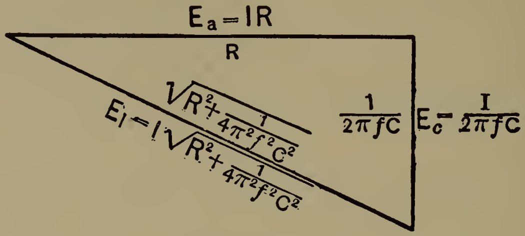

Fig. 25.

(j) The ratio of the inductance L of an alternating current circuit to its resistance R is called the *time constant* of the circuit.

$$\text{Time constant} = \frac{L}{R}. \qquad (129)$$

"The value of the *time constant* is a measure of the growth of the current" in an inductive circuit. When no inductance is present the current instantly reaches the value $\frac{E}{R}$ in which E is the impressed electromotive force, and R the resistance of the circuit. Otherwise it takes time for the maximum value to be reached proportional to $\frac{L}{R}$. But if the resistance is increased in proportion to L the time may be the same for large as for small inductances.

60. **Inductance, Capacity and Resistance. Series.** — EXAMPLE. — How many volts must an alternator with disk armature without iron generate when it is desired to have 1000 terminal voltage, the current being 25 amperes, fre-

quency 100 cycles per second, resistance 1 ohm, and coefficient of inductance $L = 0.01$ henry? — *Jackson.**

SOLUTION. — Reference to Fig. 24 and formula (120) will give for the E.M.F. of self-induction

$$E_s = 2\pi fLI = 2\pi \times 100 \times 0.01 \times 25 = 157 \text{ volts.}$$

The active pressure from (122) will be

$$E_a = IR = 25 \times 1 = 25 \text{ volts.}$$

The external required voltage is 1000 volts. Therefore the total active pressure becomes

$$E_a = 25 + 1000 = 1025 \text{ volts.}$$

Hence the impressed pressure to be developed by the alternator is from (123)

$$E_i = \sqrt{E_s^2 + E_a^2} = \sqrt{\overline{157}^2 + \overline{1025}^2} = 1037 \text{ volts.}$$

EXAMPLE. — How many more volts than in the preceding case must be generated in the armature of the above machine if iron be used in the core so that when carrying 25 amperes the inductance is 0.025 henry?

SOLUTION. — As before the total active pressure is

$$E_a = 1000 + 25 = 1025 \text{ volts.}$$

and $E_s = 2\pi \times 100 \times 0.025 \times 25 = 392.7$ volts.

Therefore the total impressed E.M.F. must be

$$E_i = \sqrt{\overline{1025}^2 + \overline{392.7}^2} = 1098 \text{ volts,}$$

or $1098 - 1037 = 61$ volts higher pressure than before.

When two or more inductances are in series, and have equal time constants, that is, if

$$\frac{L_1}{R_1} = \frac{L_2}{R_2} = \frac{L_3}{R_3} \text{ etc.,}$$

* See Jackson's *Alternating Currents*, p. 75.

then the total reactance is the sum of the individual reactances, and the total drop of potential over them is the sum of the individual drops. If the time constants are not equal the resultant impedance and potential must be found in the third side of the triangle whose other two sides are the separate impedances and potentials.

EXAMPLE. — A circuit has a resistance of 10 ohms and a capacity of 100 microfarads = 0.000100 farad. What must be the impressed E.M.F. at a frequency of $127\frac{1}{2}$ to send 10 amperes through the circuit? How many amperes will flow if the impressed pressure be 160 volts? What is the angle of lead?

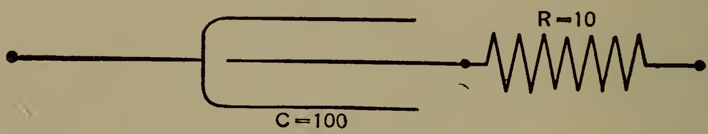

Fig. 26.

SOLUTION. — The relation may be indicated as in Fig. 26, in which C is the condenser, and R the resistance. Also referring to Fig. 25 and formula (127),

$$E_c = \frac{I}{2\,\pi f C} = \frac{10}{2\,\pi \times 127.5 \times 0.000100} = 125 \text{ volts.}$$

$$E_a = 10 \times 10 = 100 \text{ volts.}$$

Therefore, $E_i = \sqrt{\overline{100}^2 + \overline{125}^2} = 160$ volts.

$$\text{Impedance} = \sqrt{\overline{10}^2 + \overline{12.5}^2} = 16 \text{ ohms.}$$

Also I at 160 volts is

$$I = \frac{E}{R} = \frac{160}{16} = 10 \text{ amperes.}$$

The angle of lead, or negative lag, is the angle whose tangent is

$$\tan \phi = \frac{10}{12.5} = 0.8.$$

The angle corresponding to this tangent from a table of natural functions is (table 14)

$$\phi = -\ 51^\circ\ 20'.$$

EXAMPLE. — Suppose a circuit to have a negligible ohmic resistance, a capacity of 100 microfarads and an inductance of 0.01 henry in series; if the frequency is taken at $127\frac{1}{2}$ for convenience in calculation, how many volts must be applied to send 10 amperes through the circuit? If 220 volts are impressed upon the circuit how many amperes will flow?

SOLUTION. — It has already been shown that capacity and inductance act oppositely or 180° apart. Reference to Figs. 24 and 25 shows how they are represented diagramatically. We may then find the reactance due to

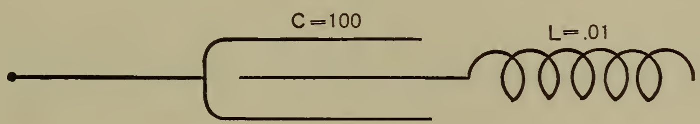

Fig. 27.

each, and plat by oppositely directed vertical lines — the inductive resistance upward from a point, and the capacity resistance downward from the same point. The difference is then the resultant reactance.

From (121)

$$R_s = 2\pi f L = 2\pi \times 127\frac{1}{2} \times 0.01 = 8 \text{ ohms}.$$

Also from (128)

$$R_c = \frac{1}{2\,\pi f C} = \frac{1}{2\,\pi \times 127\tfrac{1}{2} \times 0.0001} = 12.5 \text{ ohms.}$$

Therefore the resultant reactance is

$$R = 12.5 - 8 = 4.5 \text{ ohms.}$$

Hence from Ohm's law

$$E = IR = 10 \times 4.5 = 45 \text{ volts.}$$

Also at 220 volts the current would be

$$I = \frac{E}{R} = \frac{220}{4.5} = 48.8 \text{ amperes.}$$

It will be obvious from this that it is possible so to adjust a capacity that its resistance will be equal to the inductive resistance, and thus counteract it and make the circuit equivalent to one with neither inductance nor capacity.

When two capacities are in series the total resistance is the sum of the two resistances and the total fall of potential is the sum of the individual drops.

Example. — Derive the formula for calculating the E.M.F. of self-induction in a coil of known constants.

Solution. — The E.M.F. in any coil is proportional to the *rate* of *change* of the lines of force threading through the coil, and to the number of turns in the coil. If N be the whole change in the number of lines of force due to any cause in T seconds, and the number of turns is represented by n, the E.M.F. will be

$$E = \frac{nN}{10^8\,T}. \qquad (130)$$

In this equation $\frac{nN}{T}$ gives absolute units of E.M.F., or dynes, and 10^8 reduces to practical units of E.M.F. In a long solenoid

$$N = \frac{4\pi n' A I}{10}, \tag{131}$$

in which n' is the number of turns per centimeter of length of the solenoid, A the area of the coil, and $\frac{I}{10}$ absolute amperes, I being practical amperes. Substituting this value of N in (130) gives for the self-induced electromotive force

$$E = \frac{4\pi n n' A I}{10^9 T}. \tag{132}$$

The number of lines passing through the coil when the current is 1 C.G.S. unit is $4\pi n' A$. If the coil have iron in it so that its permeability becomes μ instead of 1, (132) will become

$$E = \frac{4\pi n n' A \mu I}{10^9 T}. \tag{133}$$

Also in (132) $4\pi n n' A = L$, the *coefficient* of *self-induction*, and we may write

$$E = \frac{LI}{10^9 T}. \tag{134}$$

Here L is expressed in absolute units. If L is given in henrys,

$$E = \frac{LI}{T}. \tag{135}$$

When (133) is applicable, $L = 4\pi nn'A\mu$ absolute units; or

$$L = \frac{4\pi nn'A\mu}{10^9} \text{ henrys.}$$

This again gives (135)

$$E = L\frac{I}{T}.$$

Therefore the *E.M.F. of self-induction is equal to the coefficient of self-induction in henrys multiplied by the rate of change of the current in amperes per second.*

Since

$$N = \frac{4\pi n'A\mu I}{10}, \text{ and } E = \frac{4\pi nn'A\mu I}{10^9 T} = \frac{LI}{10^9 T},$$

therefore we may express, for unit current and permeability,

$$L = \frac{nN}{10^9}. \qquad (136)$$

Example. — How many volts of counter E.M.F. will be developed in a solenoid 50 centimeters long uniformly wound with 250 turns of wire, the area being 4 square centimeters, and in which the current of 5 amperes takes 0.01 second to rise from zero to its maximum value?

Solution. — $n = 250$ turns, $l = 50$ cm.; hence $n' = 250 \div 50 = 5$ turns per cm. Also $A = 4$ sq. cm., $\mu = 1$, and $T = \frac{1}{100}$ second. Applying (133)

$$E = \frac{4\pi \times 250 \times 5 \times 4 \times 1 \times 5}{10^9 \times 0.01} = 0.00628 \text{ volt.}$$

Example. — Suppose that in the above example a core of iron whose permeability may be taken to be 500 were put in the solenoid How many volts of counter E.M.F. will be set up in the coil?

SOLUTION.— $E = \dfrac{4\pi \times 250 \times 5 \times 4 \times 500}{10^9 \times 0.01} = 3.14$ volts.

EXAMPLE. — Find the coefficient of self-induction of a coil of 250 turns uniformly wound on an iron ring 100 centimeters in mean circumference and having a cross sectional area of 20 square centimeters, and carrying a current varying at the rate of 2 amperes per second, μ being 250.—*Jackson.*

SOLUTION. — Since $N = \dfrac{4\pi n' A I \mu}{10}$,

$$\frac{10 \times N}{I} = 4\pi n' A\mu = 4\pi \times 25 \times 20 \times 250 = 1{,}571{,}000,$$

the number of lines per ampere per second. Therefore

$$L = \frac{nN}{10^9} = \frac{1{,}571{,}000 \times 2500}{10^9} = 3.93 \text{ henrys.}$$

Otherwise, since $$E = \frac{4\pi n n' A \mu I}{10^9},$$

where I is the number of amperes per second, and since *L is measured by the number of volts of counter E.M.F. when the current changes at the rate of 1 ampere per second,*

$$L = E = \frac{4\pi n n' A\mu}{10^9} = \frac{4\pi \times 2500 \times 25 \times 20 \times 250}{10^9}$$

$= 3.93$ henrys, as before.

61. Inductance, Capacity, and Resistance. Parallel. — EXAMPLE. — What is the resistance when 5 ohms and 15 ohms are connected in parallel? How many volts will cause 20 amperes to flow?

SOLUTION. — From (17)

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} + \frac{1}{15} = \frac{4}{15}.$$

Hence

$R = \frac{15}{4} = 3.75$ ohms, and $E = IR = 20 \times 3.75 = 75$ volts.

EXAMPLE. — Suppose two inductances with negligible resistances, $L_1 = 0.01$ and $L_2 = 0.05$, be placed in parallel on a circuit whose frequency is 63.75. Find the joint impedance and the current when the voltage is 200.

SOLUTION. — Since there are no ohmic resistances the *time constants* are equal and the same rule applies as in the last example; namely, the combined impedance is the reciprocal of the sum of the reciprocals of the individual reactances. Hence

$$\frac{1}{R} = \frac{1}{2\pi f L_1} + \frac{1}{2\pi f L_2}. \tag{137}$$

Whence

$$\frac{1}{R} = \frac{1}{2\pi \times 63.75 \times 0.01} + \frac{1}{2\pi \times 63.75 \times 0.05} = \frac{6}{20}.$$

Therefore
$$R = \frac{20}{6} = 3\tfrac{1}{3} \text{ ohms,}$$

and
$$I = \frac{E}{R} = \frac{200}{3\frac{1}{3}} = 60 \text{ amperes.}$$

EXAMPLE. — An inductance of 0.02 henry is connected in parallel with a resistance of 20 ohms. What is the impedance, and how many volts are required for 50 amperes, when the frequency is 78.6 so as to make $2\pi f = 500$?

SOLUTION. — The connections are indicated in Fig. 28. The time constants are not alike, hence we must take the *geometric* sum of the reciprocals as the reciprocal

of the required impedance. That is, the combined conductivity will be the hypotenuse of the right triangle of

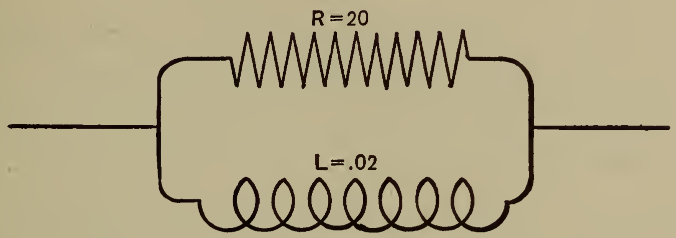

Fig. 28.

which the ohmic conductivity and the reactive conductivity are the two sides, respectively.

$$\frac{1}{R_1} = \frac{1}{20} = 0.05, \text{ and } \frac{1}{2\,\pi f L} = \frac{1}{10} = 0.10.$$

These values are then represented as in Fig. 29.

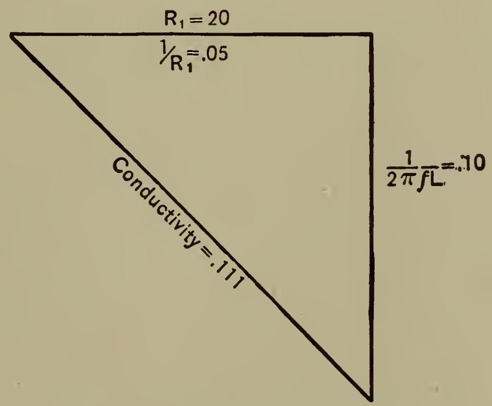

Fig. 29.

$$\frac{1}{R} = \sqrt{\left(\frac{1}{R_1}\right)^2 + \left(\frac{1}{2\,\pi f L}\right)^2} = 0.111.$$

Whence $\quad$ Impedance $R = \dfrac{1}{0.111} = 9$ ohms,

and $\quad E = IR = 50 \times 9 = 450$ volts.

EXAMPLE. — If a resistance of $1.6\frac{2}{3}$ ohms be placed in parallel with a capacity of 1000 microfarads and a pressure of 100 volts applied at a frequency of $127\frac{1}{2}$, how many amperes will flow in the circuit?

SOLUTION. — This is similar to the preceding and Fig. 30 shows the connections. We must again repre-

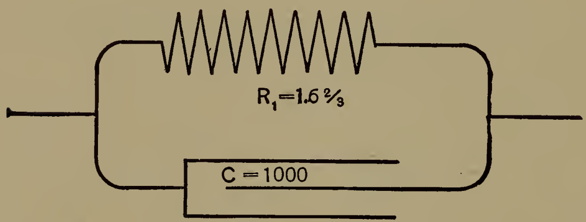

Fig. 30.

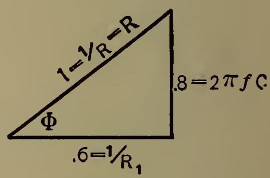

Fig. 31.

sent the conductances by the two sides of a right triangle; the hypotenuse will be the total conductance whose reciprocal will be the required impedance.

$$\frac{1}{R_1} = \frac{1}{1.6\frac{2}{3}} = 0.6,$$

and $\quad 2\pi fC = 2\pi \times 127\frac{1}{2} \times 0.001000 = 0.8.$

Whence $\quad \dfrac{1}{R} = \sqrt{\overline{0.6}^2 + \overline{0.8}^2} = 1,$ and $R = 1.$

Therefore $\quad I = \dfrac{E}{R} = \dfrac{100}{1} = 100$ amperes.

$$\text{Tangent } \phi = \frac{0.8}{0.6} = 1.333.$$

Hence $\quad \phi = 53° \, 8''$, *lead angle.*

Example. — A capacity of 50 microfarads is connected in parallel with an inductance of 0.05 henry; when the frequency is 127.5 how many volts will be required for 10 amperes?

Solution. — Capacity and inductance lines are opposed to each other and are drawn vertically. Fig. 32 gives the connections. The conductances are

$$2\,\pi f C = 0.04, \text{ and } \frac{1}{2\,\pi f L} = 0.025,$$

Therefore $$\frac{1}{R} = \sqrt{\overline{0.04}^2 + \overline{0.025}^2} = 0.0149,$$

and $$R = \frac{1}{0.0149} = 67 \text{ ohms.}$$

Also $$E = IR = 10 \times 67 = 670 \text{ volts.}$$

Example. — There are in parallel two circuits as follows: the first having $R_1 = 8$ ohms and $L_1 = 0.0075$ henry; the second having $R_2 = 12.5$ ohms and $L_2 = 0.0125$; if the frequency is 127.5, what is the total impedance?

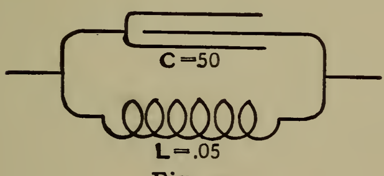

Fig. 32.

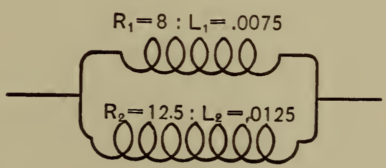

Fig. 33.

Solution. — Connections are represented in Fig. 33. First find

$$R' = \sqrt{R_1^2 + \overline{2\,\pi f L_1}^2} = \sqrt{8^2 + 6^2} = 10 \text{ ohms.}$$

$$R'' = \sqrt{R_2^2 + \overline{2\,\pi f L_2}^2} = \sqrt{\overline{12.5}^2 + \overline{10}^2} = 16 \text{ ohms.}$$

These are obtained graphically as in Fig. 34. The conductances are $\frac{1}{10} = 0.1$, and $\frac{1}{16} = 0.0625$ respectively. Now represent these on some convenient scale, say one inch for 0.05, or 2 inches for 0.1, and draw them at the proper angles as in Fig. 35. Lay down OL hori-

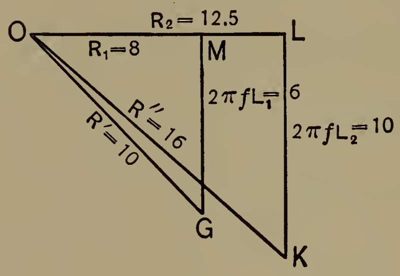

Fig. 34.

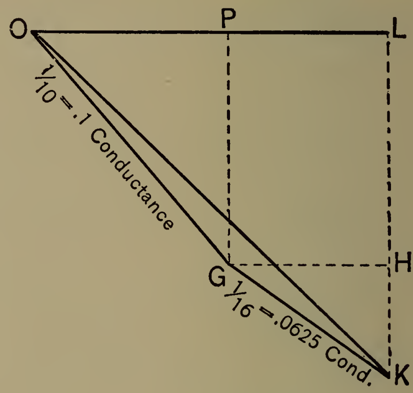

Fig. 35.

zontally, then $OG = 0.1$, making the same angle with OL, as OG in Fig. 34. From G draw GK in the same direction as OK and make it equal to 0.0625. Then draw OK and measure it; then from the scale taken determine its length. We find it to be 0.1625. The impedance is its reciprocal, or 6.15 ohms.

Example. — A non-inductive resistance of 5 ohms is in series with an inductive resistance of 5 ohms and 0.02 henry. What impressed pressure at a frequency of 100 is necessary to furnish 25 amperes?

Solution. — As before find the impedance of the inductive path. Both from Fig. 36 and (123),

$$R = \sqrt{R_2^2 + \overline{2\pi fL}^2} = \sqrt{5^2 + \overline{2\pi \times 100 \times 0.02}^2} = 13.57 \text{ ohms.}$$

The conductance of the reactive branch is then $\frac{1}{13.57}$ = 0.0736, which is platted to scale in Fig. 37, having the same direction as LK in Fig. 36. From K in Fig. 37 KP is drawn $= \frac{1}{5} = 0.2$, the conductance of the non-inductive branch. LP is then drawn and measured, and from the scale chosen is found to be 0.261, the total conductance. The reciprocal gives the impedance = 3.83 ohms. This is also obtained from the trigonometric relations. Hence for 25 amperes the pressure in volts will be

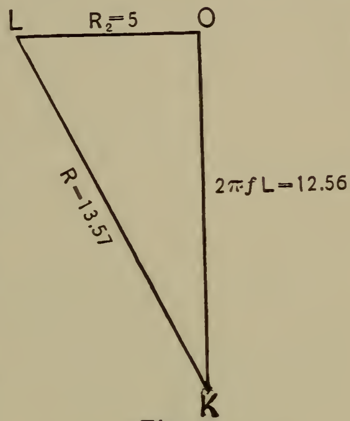

Fig. 36.

$$E = 25 \times 3.83 = 95.75 \text{ volts.}$$

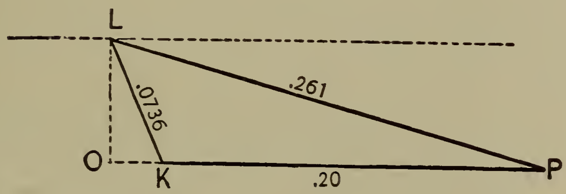

Fig. 37.

62. **Problems.** — 1. Find the *average* and the *effective* E.M.F. of an alternator whose maximum E.M.F. is known from its curve to be 1500 volts.

Average = 955.5 volts.
Effective = 1060.5 volt.

2. If a flat coil of wire develops 5 volts when its plane is parallel with the lines of force of the magnetic field, what will be the E.M.F. at the instant when its plane stands at an angle of 45° with respect to the lines of force?

$$\text{Required } E : 5 \text{ volts} :: \sin 45^\circ : \sin 90^\circ, \text{ or}$$
$$E : 5 \text{ volts} :: \tfrac{1}{2}\sqrt{2} : 1$$
$$E = \tfrac{5}{2}\sqrt{2} = 3.5 \text{ volts.}$$

3. How many amperes of current will flow in a circuit under a dynamo pressure of 1200 volts, when the combined resistance of all the circuit is 20 ohms and the average coefficient of self-induction is 0.0312 henry, frequency 127.5? $I = 37.5$ amperes.

4. A dynamo having 8 poles makes 2200 r.p.m. It has a resistance of 0.1 ohm and an inductance of 0.01 henry. What E.M.F. must it generate to give a terminal pressure of 1000 volts if the current is 25 amperes? Draw proper diagrams.

$$E_s = 2\,\pi f L I = 230 \text{ volts.}$$
$$E_a = IR = 2.5 \text{ volts.}$$
$$\text{Therefore, } E_i = \sqrt{(1000 + 2.5)^2 + (230)^2}$$
$$= 1028.6 \text{ volts.}$$

5. If an 8-pole machine is to run at 1800 r.p.m and has a coefficient of self-induction of 0.005 henry, resistance of 2 ohms, what voltage must be generated in the armature winding so that a terminal voltage of 1000 may be furnished when the current is 25 amperes?

$E_i = 1054.2$ volts.

6. The pressure applied to a circuit whose resistance was 5 ohms was 250.32 volts, the circuit carrying 50 amperes; what was the E.M.F. of self-induction? Also if the frequency was 63.75, what was the coefficient of inductance?

$E_s = 12.65$ volts.
$L = 0.00063$ henry.

7. How many amperes will pass through a circuit under 100.7 volts applied at a frequency of 63.75, when the resistance is 10 ohms and inductance 0.01 henry? What are the ohmic and inductive drops?

$I = 10$ amperes.
$E_a = 100$ volts.
$E_s = 40$ volts.

8. What resistance may be added in a circuit whose inductive drop at 100 amperes is 300 volts, when 500 volts is the impressed pressure? What is the active pressure and impedance?

$R_1 = 4$ ohms.
$E_a = 400$ volts.
$R = 5$ ohms.

9. What E.M.F. at a frequency of 125 must be applied to a condenser circuit whose capacity is 100 microfarads, so that 10 amperes of current will flow?

$E_i = 127.4$ volts.

10. What capacity must be put into an alternating current circuit of negligible resistance, so that 50 amperes may be obtained at a pressure of 500 volts and frequency of $127\frac{1}{2}$?

$C = 125$ microfarads.

11. What is the capacity reactance in problem 10?

$R = 0.1$ ohm.

12. What is the impedance of a circuit which contains an ohmic resistance of 40 ohms in series with a capacity resistance of 30 ohms?

$$R = \sqrt{30^2 + 40^2} = 50 \text{ ohms.}$$

13. If 10 amperes are required through the circuit in problem 12, how many volts of E.M.F. must be impressed upon it? Find the ohmic and capacity drops.

$$E_i = 10 \times 50 = 500 \text{ volts.}$$
$$E_c = 10 \times 30 = 300 \text{ volts.}$$
$$E_a = 10 \times 40 = 400 \text{ volts.}$$

14. A dynamo supplies 25 amperes to a circuit consisting of 10 ohms in series with a capacity of 100 microfarads at a frequency of 63.75. What is the terminal voltage of the dynamo, and what is the drop in each portion of the circuit?

$$E_i = 673 \text{ volts.}$$
$$E_a = 250 \text{ volts.}$$
$$E_c = 625 \text{ volts.}$$

15. When a capacity of 10 microfarads is in series with a resistance of 10 ohms, and an E.M.F. of 638 volts at a frequency of 100 is impressed upon the circuit, how many amperes will pass through it, and what will be the drop of potential on each portion of the circuit?

$$I = 4 \text{ amperes.}$$
$$E_a = 40 \text{ volts.}$$
$$E_c = 636.8 \text{ volts.}$$

16. What is the time constant of a circuit whose resistance is 10 ohms and whose inductance is 0.05 henry?

$$t = \frac{L}{R} = \frac{0.05}{10} = 0.005 \text{ sec.}$$

17. What is the coefficient of self-induction of a circuit whose time constant is 0.01 second and whose resistance is 5 ohms. $L = 0.05$ henry.

18. An alternating current circuit is carrying 5 amperes under an impressed pressure of 50 volts at a frequency of $127\frac{1}{2}$. If its resistance is 6 ohms, what is its time constant?

$$t = \frac{L}{R} = 0.0016\tfrac{2}{3} \text{ seconds.}$$

19. What current will flow through a circuit having a negligible resistance, an inductance of 0.05 henry, and a capacity of 1 microfarad in series, when a pressure of 500 volts is applied at a frequency of 100? Draw diagram. $I = 0.32$ ampere.

20. What capacity must be put in series with an inductance of 0.02 henry when the frequency is 63.75 so that the impedance shall be zero?

$C = 312$ microfarads.

21. How many amperes will flow under the conditions given in problem 20, if a resistance of 50 ohms be added in series and 1000 volts E.M.F. be applied?

$I = 20$ amperes.

22. What is the coefficient of self-induction of a coil of 500 turns through which the magnetism is changing at the rate of 10^5 lines of force per second?

$$L = nN \div 10^9 = 0.05 \text{ henry.}$$

23. How many lines of force are produced in a coil having 10 turns per centimeter of its length and a cross

section of 10 square centimeters when it carries 10 amperes of current, no iron being in its vicinity?

$$N = \frac{4\pi n' A I}{10} = 1256 \text{ lines.}$$

24. How would the number of lines be affected if an iron core were introduced into the coil, and its permeability be given as $\mu = 500$?

$$N = \frac{4\pi n' A I \mu}{10} = 628{,}000 \text{ lines.}$$

25. If the total length of the coil in problem 24 is 50 centimeters, what E.M.F. of self-induction is set up under the conditions named?

$$E = \frac{4\pi n n' A \mu I}{10^9} = 3.14 \text{ volt.}$$

26. What is the inductance under the conditions named in problem 25?

$$L = \frac{E}{I} = \text{in absolute value.}$$

$$L = \frac{E}{I} = \frac{4\pi n n' A \mu}{10^9} = 0.314 \text{ henry,}$$

in practical units.

27. If a voltage of 1040 at a frequency of 125 be applied to a circuit consisting of a resistance of 10 ohms, an inductance of 0.05 henry, and a capacity of 100 microfarads, all in series, how many amperes of current will flow in the circuit, and what is the impedance? Also the drop over each portion of the circuit?

$I = 36.7$ amperes.
$R = 28.3$ ohms.
$E_a = 367$ volts.
$E_c = 466$ volts.

28. There are in parallel two inductances, $L_1 = 0.0002$ henry, $L_2 = 0.0004$ henry. The frequency is 100. What is the impedance, and how much current will flow under an impressed pressure of 8.37 volts?

$R = 0.0837$ ohm.
$I = 100$ amperes.

29. An inductance $L = 0.05$ henry is placed in parallel with a non-inductive resistance of 10 ohms. When a voltage of 177 is applied to the circuit at a frequency of 60, how many amperes will flow? $R = 8.85$ ohms.
$I = 20$ amperes.

30. Let an inductance of 0.01 henry be placed in parallel with a capacity of 50 microfarads; what E.M.F. at a frequency of $127\frac{1}{2}$ must be applied to the circuit to give 10 amperes of current? How many amperes will pass in each branch? $R = 11.77$ ohms.
$E = 117.7$ volts.
$I_s = 14.71$ amperes.
$I_c = 4.71$ amperes.

31. A coil of wire has an ohmic resistance of 10 ohms and an inductance of 0.0156 henry. What is the impedance when the frequency is $127\frac{1}{2}$?

$R = 16$ ohms.

32. A capacity circuit has a capacity of $156\frac{1}{4}$ microfarads in series with a resistance of 10 ohms. Find the impedance at a frequency of $127\frac{1}{2}$. $R = 12.8$ ohms.

33. Place the circuit of problem 31 in parallel with that of problem 32 and find the joint impedance. Find the

number of amperes of current when 787 volts of E.M.F. are applied.

$R = 7.87$ ohms.
$I = 100$ amperes.
$\phi = 19^\circ\ 42'$.

SOLUTION. — $R_s = \sqrt{10^2 + 12.5^2} = 16$ ohms.

$$R_c = \sqrt{10^2 + 8^2} = 12.8 \text{ ohms.}$$

$$\frac{1}{R_s} = \frac{1}{16} = 0.0625.$$

$$\frac{1}{R_c} = \frac{1}{12.8} = 0.078.$$

In the second diagram, Fig. 38, plat conductances 0.078 and 0.0625 parallel to their respective resistances, then complete the triangle whose third side will to scale represent the total conductance. Hence

$$\frac{1}{R} = 0.127,$$

Whence $\qquad R = \dfrac{1}{0.127} = 7.87$ ohms.

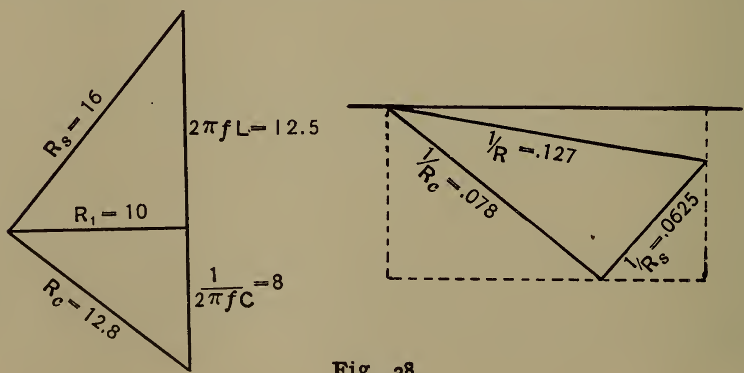

Fig. 38.

34. An inductive resistance of 8 ohms and coefficient of self-induction of 0.015 henry is connected in parallel

with a circuit having 8 ohms resistance and $416\frac{2}{3}$ microfarads capacity. Find the joint impedance and the current flowing when the impressed E.M.F. is 1350 volts, the frequency being 63.75.

$$R = 6.25 \text{ ohms.}$$
$$I = 200 \text{ amperes.}$$
$$\phi = 0^\circ.$$

SOLUTION.—$R_s = 10$ is the impedance of the first circuit, $R_c = 10$ is the impedance of the second. The conductances corresponding are each 0.1. These are represented to scale by $A\ L$ and $L\ B$, Fig. 39, drawn to

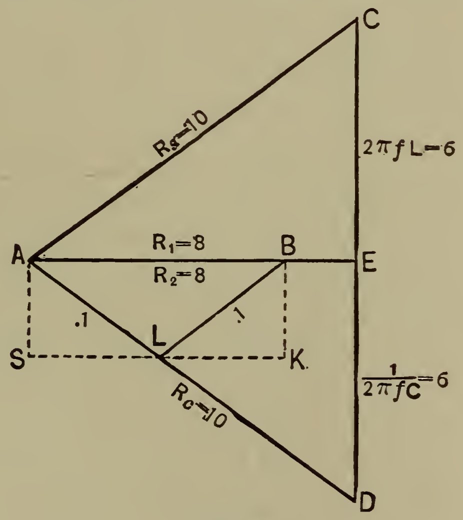

Fig. 39.

each other at the same angle as $A\ D$ and $A\ C$. The resultant is $A\ B$ with no lag. $A\ B = 0.1599$, the reciprocal of which is 6.25 ohms, the desired total impedance.

Therefore $$I = \frac{E}{R} = \frac{1350}{6.25} = 200 \text{ amperes.}$$

35. Find the joint impedance when an inductance of 0.02 henry is in parallel with a capacity of 312.5 microfarads, the ohmic resistance being negligible in each case, and the frequency is 63.75. $R =$ infinity.

36. How much current would flow in the circuit if 100 ohms were put in parallel with the combination in problem 35, and 1000 volts be applied? $I = \frac{1000}{100} = 10$ amps.

37. A non-inductive resistance of 50 ohms is in parallel with a capacity of 200 microfarads. Find the joint impedance. Also now place an inductance of 0.015 henry in series with the parallel combination, and find the total impedance and the E.M.F. necessary for 10 amperes when the frequency is given as 100. $R_c = 7.87$ ohms.
$R = 2.08$ ohms. $\phi = 7°28'$. $E = 20.8$ volts.

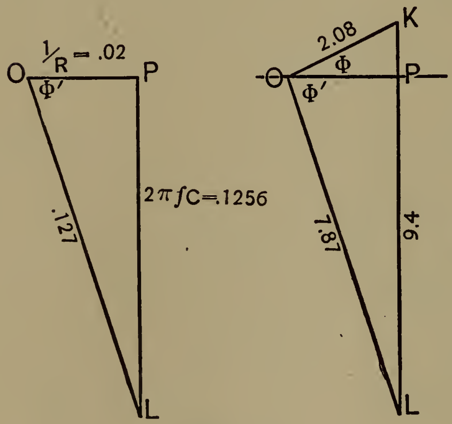

Fig. 40.

SOLUTION. — Joint impedance of parallel part is obtained by finding the third side of the triangle. $\frac{1}{R} = \frac{1}{50} = 0.02$ forming base and $2\pi fC = 6.28 \times 100 \times 0.0002 = 0.1256$ forming the perpendicular. The hypotenuse is 0.127. Its reciprocal $= 7.87$ ohms. Now lay the latter down to scale and at the same angle with the horizontal as OL and from L draw upwards to scale $LK = 2\pi fL = 6.28 \times 100 \times 0.015 = 9.4$ ohms. Then $OK = 2.08$, the joint impedance; $\phi = 7°28' =$ angle lag.

XIII.

ALTERNATING CURRENT DISTRIBUTION.

63. **Alternating Current Circuits.**—It has already been shown that there are several causes making it inaccurate to apply Ohm's law strictly to calculations in alternating currents. However, where the transformer secondaries are loaded with incandescent lamps, the effect of inductance becomes negligibly small, so that the inaccuracies will not be very great for moderate lengths of circuits.

Practically the *ratio* of *transformation* of transformers is the ratio of the primary turns to the secondary turns, or the ratio of the secondary current to the primary current. The ratio of transformation of transformers used for incandescent lighting on single-phase circuits is usually 20 or 10. In the monocyclic * system ratios of 9 and 4 are often used. In long distance polyphase transmission various ratios are employed, particularly in the large sizes of transformers. In a town having a population of 5000 to 10,000, the lighting plant being centrally located, the primary E.M.F. will be, perhaps, 2000 to 2200 volts, and the secondary 100 to 110 volts, the ratio being 20. Where the primary mains are quite short, possibly the primary E.M.F. would be 1000 volts, and the ratio of transformation 10. In the Niagara-Buffalo line, the power is brought to the city limits at 22,000 volts, then transformed to 11,000, the ratio of transformation being in this case but 2.

* The monocyclic system is no longer a commercial system.

The efficiency of the larger sizes of transformers is from 95 per cent to 99 per cent, and the *core losses* and the *copper losses* are about equal. To obtain a given energy in the secondary, the primary current will be increased by the amount of the core loss per cent, while the primary voltage will be increased by the per cent of copper loss.

EXAMPLE. — When the secondary of a transformer is wound for 52 volts, and the ratio of transformation is 20, what is the primary current for a 1000 watt transformer?

SOLUTION. — Secondary current

$$I_2 = 1000 \div 50 = 20 \text{ amperes.}$$

This will furnish 20 lights, 16 c.p.

$$\text{Primary voltage} = 52 \times 20 = 1040 \text{ volts.}$$

$$\text{Primary current } I_1 = 20 \div 20 = 1 \text{ ampere.}$$

Therefore,

$$\text{Watts in primary for no loss} = 1040 \times 1 = 1040 \text{ watts.}$$

If the loss be 5 per cent, then

$$\text{Watts in primary coil} = 1040 \div 0.95 = 1095.$$

Suppose copper and iron losses equal, each being $2\frac{1}{2}$ per cent. Then the primary current is

$$I_1 = 1 \div 0.975 = 1.025 \text{ amperes.}$$

Primary E.M.F.

$$E_1 = 1040 \div 0.975 = 1066.66 \text{ volts.}$$

Otherwise, $E_1 = 1095 \div 1.025 = 1067$ volts.

Strictly, the ratio of transformation here is $20 \div 1.025 = 19.4$ instead of 20. Hence the transformer * losses have the effect of reducing the ratio of transformation.

* This refers to the iron losses.

Example. — What is the maximum per cent drop in the secondaries in the last example if 50 volt, 1 ampere lamps are to be used? What size of wire will be used, and what must be the size of the primary mains, neglecting drop due to inductance, if an ohmic loss of 10 per cent be allowed, and the distance be $\frac{1}{2}$ mile?

Solution. — Secondary loss = 2 volts ÷ 52 = 3.8 per cent. Assume the distance to the lamps to be 100 feet. Then the line resistance per 1000 feet is

$$R_2 = (2 \div 20) \times \frac{1000}{200} = 0.5 \text{ ohm, which is No. 7 A.W.G.}$$

For the primary, $I_1 = 1.025$ amperes;

$$E \text{ (drop)} = \frac{1067}{0.90} - 1067 = 119 \text{ volts.}$$

Hence $R_1 = 119 \div 1.025 = 116$ ohms for 5280 feet.

Making per 1000 feet, $R_1 = 116 \div 5.28 = 22$ ohms, or No. 23 wire. This loss, of course, is excessive.

Machine voltage = 1067 ÷ 0.90 = 1186 volts.

The table No. 10 will give the impedance, the ohmic resistance and the inductive resistance in ohms per mile of those sizes in which there will be an appreciable effect at the given distances of the wires apart. These values are given for frequencies of 60 and 125 per second. The column headed R_1 means ohms resistance per mile, R_s is the inductive resistance in ohms per mile, and R is the impedance or resultant resistance in ohms per mile. It will be observed that the values in the columns headed R are obtained from

$$R = \sqrt{R_1^2 + 4\pi^2 f^2 L^2} = \sqrt{R_1^2 + R_s^2}.$$

Thus for No. 0 at 12 inches apart,

$$R = \sqrt{0.519^2 + 0.55^2} = 0.756.$$

It is safer to add 15% to the tabular inductive resistances to compensate for distortion from the sine wave.

EXAMPLE. — By means of the table determine the E.M.F. necessary to apply to a circuit $1\frac{1}{2}$ miles long to carry 10 amperes, when No. 1 wire is used 12 inches apart, and the frequency is 60.

SOLUTION. —

R_1 of No. 1 for 3 miles $= 0.655 \times 3 = 1.965$ ohms.

R_s of No. 1, 3 miles $= 0.565 \times 3 = 1.695$ ohms.

Impedance $R = 0.865 \times 3 = 2.595$ ohms,

or $R = \sqrt{1.695^2 + 1.965^2} = 2.595$ ohms.

Since $E = IR$, $I = 10$, and $R = 2.595$,

$E = 10 \times 2.595 = 25.95$ volts line drop.

If there were no self-induction E would be

$$1.965 \times 10 = 19.65 \text{ volts drop.}$$

Therefore $25.95 - 19.65 = 6.30$ volts necessary for inductive drop on line.

At 125 cycles per second this would be

$$E = 1.349 \times 3 \times 10 = 40.47 \text{ volts total drop.}$$

Inductive drop is $40.47 - 19.65 = 20.82$ volts. If the load is non-inductive, as incandescent lamps, the generator must furnish 20.82 volts more on account of line self-induction.

10. ALTERNATING CURRENT WIRING TABLE.*

Gauge Number A.W.G.	Res. in Ohms per Mile.	Inductive Resistance and Impedance in Ohms per Mile at 60 Cycles per Sec.						Inductive Resistance and Impedance in Ohms per Mile at 125 Cycles per Sec.					
		12″ on Centers.		18″ on Centers.		24″ on Centers.		12″ on Centers.		18″ on Centers.		24″ on Centers.	
	R_1	R_s	R	R_s	R	R_s	R	R_s	R	R_s	R	R_s	R
0000	0.259	0.508	0.570	0.557	0.615	0.595	0.646	1.060	1.092	1.170	1.190	1.230	1.260
000	0.324	0.523	0.616	0.573	0.658	0.607	0.686	1.090	1.138	1.200	1.237	1.260	1.305
00	0.412	0.534	0.682	0.588	0.725	0.618	0.749	1.120	1.194	1.230	1.297	1.290	1.357
0	0.519	0.550	0.756	0.603	0.796	0.633	0.818	1.150	1.258	1.260	1.360	1.320	1.415
1	0.655	0.565	0.865	0.614	0.896	0.648	0.920	1.180	1.349	1.280	1.436	1.350	1.500
2	0.826	0.580	1.008	0.629	1.038	0.663	1.060	1.210	1.466	1.310	1.550	1.380	1.610
3	1.041	0.591	1.196	0.644	1.223	0.674	1.240	1.240	1.610	1.340	1.700	1.410	1.750
4	1.313	0.606	1.448	0.656	1.467	0.690	1.480	1.260	1.820	1.370	1.890	1.440	1.940
5	1.656	0.620	1.760	0.670	1.780	0.704	1.800	1.300	2.100	1.400	2.170	1.470	2.220
6	2.088	0.633	2.180	0.685	2.200	0.720	2.710	1.320	2.460	1.430	2.510	1.490	2.560
7	2.633	0.647	2.710	0.700	2.720	0.730	2.730	1.350	2.930	1.460	3.000	1.520	3.040
8	3.320	0.662	3.380	0.712	3.390	0.742	3.400	1.380	3.590	1.480	3.630	1.550	3.660
9	4.186	0.677	4.210	0.727	4.220	0.761	4.230	1.410	4.390	1.510	4.430	1.580	4.450
10	5.280	0.688	5.320	0.742	5.330	0.776	5.340	1.440	5.470	1.540	5.500	1.620	5.530

* Adapted from Emmet, *Alternating Current Wiring and Distribution.*

Let the load be 20 groups of 110 volt incandescent lamps, 5 in series in each group. This will require $110 \times 5 = 550$ volts at the lamp terminals. The resistance of the load is therefore $\frac{5 \times 220}{20} = 55$ ohms. This is in series with the line resistance and inductance. Take the frequency at 60, then neglecting dynamo resistance and inductance, the impedance of the circuit will be

$$R = \sqrt{\overline{1.965 + 55}^2 + \overline{1.695}^2} = 57 \text{ ohms.}$$

The total impressed E.M.F. must be

$$E = IR = 10 \times 57 = 570 \text{ volts.}$$

Hence the line drop due to all causes is $570 - 550 = 20$ volts, while the ohmic drop alone is 19.65 volts. Therefore it is plain that for non-inductive loads and very small machine inductance, the line self-induction is not a very disturbing factor.

Example. — Assume the machine resistance to be 0.4 ohm and the coefficient of self-induction $L = 0.02$ henry, speed so as to make $f = 60$ cycles, supplying 10 amperes as the above load. Find the total impedance and the E.M.F. in the armature conductors.

Solution. — Total reactance is $R_s = 1.695 + 2\pi \times 60 \times 0.02 = 9.231$ ohms. The total impedance is

$$R = \sqrt{\overline{56.965}^2 + \overline{9.231}^2} = 58 \text{ ohms.}$$

This is not appreciably different from that found above because the resistance is quite large in proportion to the

reactance; that is, the time constant is very small. Hence the armature self-induction has not seriously disturbed the regulation.

EXAMPLE. — It is required to determine the size of wire at 12 inches apart for the circuit and load given above so that the total drop shall not exceed 25 volts.

SOLUTION. — E.M.F. at load = 550 volts.
Impressed E.M.F. = 550 + 25 = 575 volts.
Armature reactance $R_s = 7.536$ ohms $= 2\pi \times 60 \times 0.02$.
Let the line resistance be R_1, then the impedance

$$R = \sqrt{R_1^2 + \overline{x + 7.536}^2} = \tfrac{575}{10} = 57.5 \text{ ohms.}$$

Solving for R_1 and neglecting terms containing x, since the line reactance is relatively small, we get

$$R_1 = 57.4 \text{ ohms.}$$

Therefore the line resistance per mile is

$$\frac{57.4 - 55}{3} = 0.823 \text{ ohm.}$$

According to the table this corresponds to No. 2 wire.

Taking into account the transformer inductance and the transformer and lamp losses under ordinary conditions, when the transformers cannot be counted on as more than three-fourths loaded, it may be assumed without great error that the self-induction will cause a lag of 8° or less, making the power factor 0.99 and the inductance factor 0.14. In case of very light loads on transformer secondaries, the figures may become in practice 0.98 for the power factor and 0.19 for the inductance factor, and 11° for the angle of lag.

Where transformers supply motor loads the power factor would probably run down to 75% or 80%. However, if both transformers and motors could be fully loaded — a rare thing in practice — the power factor with modern appliances might go over 90%.

Example. — Taking a circuit whose power factor is 99%, what is the proportion of impressed E.M.F. to active E.M.F. in the circuit?

Solution. —

$$\frac{E_i}{E_a} = \frac{\text{Impedance}}{\text{Resistance}} = \frac{\sqrt{\overline{99}^2 + \overline{14}^2}}{99} = \frac{100}{99}, \text{ approximately.}$$

In other words, when the E.M.F. necessary to force the desired current through the given resistance would be 990 volts, the impressed voltage must be 1000, 10 volts being required to overcome the inductive resistance.

Example. — Taking 0.80 as an average power factor for motor loads, the inductance factor will be about 0.595 and the lag $36\frac{1}{2}°$. Find the impedance factor and ratio of impressed volts to active E.M.F.

Solution. —

$$\text{Impedance} = \sqrt{\overline{0.80}^2 + \overline{0.595}^2} = 0.997 \text{ ohm.}$$

$$\frac{E_i}{E_a} = \frac{0.997}{0.80} = 1.246.$$

In words, if 1000 volts would send the desired current through the given resistance, 1246 volts must be impressed upon the circuit; 1246 — 1000 = 246 volts extra are required, due to self-induction.

The following examples have been taken from Emmet's "Alternating Current Wiring and Distribution." They will serve to illustrate the type of solution which may be applied to problems under the present section. In such problems as these that follow consider the ratio of transformation one, and use the primary voltage and current on both sides of the transformer.

EXAMPLE. — Assume 500 incandescent lamps of 57.5 watts each on secondaries of transformers of different sizes, half loaded, on the average. The mains to the transformers consist of No. 2 B. & S. wire 2 miles long and 18 inches apart. The frequency is 125. The lamp voltage is to be 100 and the ratio of transformation is 10. Determine the voltage required at the generator and the ohmic drop on the line.

SOLUTION. —

Lamp active volts = 1000; lamp current 28.75.

Secondary ohmic drop 3% = 30 volts; inductive drop 3% = 30 volts.

Transformers, ohmic drop 1% = 10 volts; inductive drop $12\frac{1}{2}\%$ = 130 volts.

Core loss 5% of 28.75 = 1.44 amperes. Total I = 30.19.

Line current = 30.19 amperes. Ohmic drop = $(0.826 \times 4) \times 30.19 = 99$ volts; inductive drop $(1.31 \times 4) \times 30.19 + 15\% = 182$ volts.

Now add active volts and ohmic drops for total active volts.

Whence $E_a = 1000 + 30 + 10 + 99 = 1139$ volts.

Also add inductive drops giving

$$E_s = 30 + 130 + 182 = 342 \text{ volts.}$$

Impressed pressure

$$E_i = \sqrt{1139^2 + 342^2} = 1188 \text{ volts.}$$

The line loss due to ohmic resistance = 99 volts.

Total line loss = 30 + 10 + 99 = 139 volts;

or $$1139 - 1000 = 139.$$

Example. — From a water power it is desired to run a circuit 14 miles to supply a compact district with 500 K.W. in incandescent lamps, the current being distributed in the town on the three-wire, low-tension system from a transformer substation. Station pressure is 130 volts at full load, lamps 120 volts. The maximum potential is about 10,000 volts. For the substation 40 K.W. transformers are used in seven pairs, ratio 10 to 1, primaries connected two in series, and their secondaries connected for the three-wire system. The frequency is 60. The line is No. 00 B. & S. If we may assume a power factor of 0.99, and an inductance factor of 0.14, what will be the values corresponding to those obtained in previous example?

Solution. — First reducing secondary circuits to main line conditions, we obtain 9100 volts and 59.7 amperes. By use of the table we obtain the line resistance 0.412 ohm per mile, inductive resistance at 12 inches apart 0.534 ohm per mile; and adding 15% this becomes 0.614 ohm per mile.

On secondaries,

$$0.99 \times 9100 = 9000 \text{ active volts.}$$

Also $0.14 \times 9100 = 1270$ inductive volts.

$$I = 59.7 \text{ amperes.}$$

Reducing transformers,

Ohmic drop 1% of 9100 = 91 volts.
Inductive drop 6% of 9100 = 546 volts.
Core loss, say 3% of 59.7 = 1.78 amperes.

Main line,

Ohmic drop $0.412 \times 28 \times (59.7 + 1.78) = 710$ volts.

Inductive drop

$$0.614 \times 28 \times (59.7 + 1.78) = 1060 \text{ volts.}$$

Total active volts

$$E_a = 9000 + 91 + 710 = 9801.$$

Total inductive drop

$$E_s = 1270 + 546 + 1060 = 2876 \text{ volts.}$$

Therefore the voltage to be supplied at the secondaries of the step up transformers is

$$E_i = \sqrt{9801^2 + 2876^2} = 10{,}200 \text{ volts.}$$

The current supplied by step up transformers is

$$I = 59.7 + 1.78 = 61.5 \text{ amperes.}$$

Ohmic drop 1% of 10,200 = 102 volts.

Inductive drop 6% of 10,200 = 612 volts.

Core loss 3% of 61.5 = 1.85 amperes.

Therefore the active voltage supplied by the generator is $E_a = 9801 + 102 = 9903$ volts,

And $E_s = 2876 + 612 = 3488$ volts.

Therefore $E_i = \sqrt{\overline{9903}^2 + \overline{3488}^2} = 10{,}500$ volts.

And $I = 61.5 + 1.85 = 63.4$ amperes.

Hence the generator delivers $10{,}500 \times 63.4 = 665{,}700$ *volt-amperes*, and $10{,}500 \times 59.7 = 626.8$ K.W.

The power delivered at the transformer station is $\frac{500}{120} \times 130 = 541$ K.W., and the efficiency of the plant from generator to step-down secondaries is $541 \div 627 = 86\%$.

Sixteen of the 40 K.W. transformers connected two in series to the generator, and two in series to the line are used as step-up transformers, receiving 1050 volts and delivering 10,500 volts.

The generator capacity is 665 K.W., and allowing 92% efficiency, there will be required in water power $665 \div 0.92 = 685$ K.W. Total efficiency is then 73%.

64. Formulæ and Tables for Alternating Current Wiring. — For most practical cases the following formulæ based on Ohm's law will be very convenient and sufficiently accurate for the calculation of transmission circuits. The constants which are introduced in the formulæ have the values under the different conditions given in the proper tables; these are taken from the publications of the General Electric Company.

Symbols. — I = total line current.
E = E.M.F. at customer's end of circuit.
W = Watts delivered to the customer.
% = Per cent of W loss in line.
D = Distance of transmission.

$K =$ Factor depending on the power factor and system used; given in table; equals 2160 for single phase, 100% power factor.

$T =$ 1 for continuous currents, and depends on power factor and nature of system used; given in table.

$M =$ Factor depending on frequency, size of wire and power factor; equals 1 for continuous currents; given in table.

$A =$ Factor based on 0.00000302 lb. as weight of 1 mil-foot.

Formulæ. —

$$I = T \times \frac{W}{E}. \qquad (138)$$

Area, circular mills, $$d^2 = K \times \frac{WD}{E^2 \times \%}. \qquad (139)$$

Volts line loss $$= M \times \frac{E \times \%}{100}. \qquad (140)$$

Pounds copper $$= \frac{A \times W \times K \times D^2}{E^2 \times \% \times 10^6}. \qquad (141)$$

For continuous currents,

$T = 1$, $M = 1$, $A = 6.04$, $K = 2160$.

11. Table of Wiring Constants.

System.	Values of T. Per Cent Power Factor.				Values of K. Per Cent Power Factor.					Values of A.
	95	90	85	80	100	95	90	85	80	
Single-phase	1.052	1.11	1.17	1.25	2160	2400	2660	3000	3380	6.04
Two-phase, 4-wire . .	.526	.555	.588	.625	1080	1200	1330	1500	1690	12.08
Three-phase, 3-wire . .	.607	.642	.679	.725	1080	1200	1330	1500	1690	9 06

To find the value of K for any other power factor, use the following: —

For single-phase, $$K = \frac{2160}{(\text{Power factor})^2}. \quad (142)$$

For two-phase, 4-wire, $$K = \frac{2160}{2\,(\text{Power factor})^2}. \quad (143)$$

For three-phase, 3-wire, $$K = \frac{2160}{2\,(\text{Power factor})^2}. \quad (144)$$

Example. — Suppose it is required to find K for 3-phase lines for a power factor of 75, say a circuit largely of motors partly loaded, —

$$K = \frac{2160}{2\,(.75)^2} = 1920.$$

Calculation of Circuits by the Formulæ. — Direct Current, Two-Wire Circuits.

Example. — Determine the size of wire and the loss of E.M.F. to supply 1000 lamps, 110 volts, at a distance of 800 feet from the generator, loss 10%.

Solution. —

$$d^2 = \frac{K \times W \times D}{E^2 \times \%} = \frac{2160 \times (1000 \times \frac{1}{2} \times 110) \times 800}{110^2 \times 10}$$

$$= 785{,}450 \text{ cir. mils.}$$

$$\text{Volts lost} = \frac{M \times E \times \%}{100} = \frac{1 \times 110 \times 10}{100} = 11 \text{ volts.}$$

$$\text{Weight of copper} = \frac{A \times W \times K \times D^2}{E^2 \times \% \times 10^6}$$

$$= \frac{6.04 \times (1000 \times \frac{1}{2} \times 110) \times 2160 \times \overline{800}^2}{\overline{110}^2 \times 10 \times 10^6} = 3795 \text{ lbs.}$$

12. Table of Wiring Constants.

Values of M, Wires 18 Inches Apart.

Gauge Number, B. & S. or A. W. G.	25 Cycles. Power Factor. 95	90	85	80	40 Cycles. Power Factor. 95	90	85	80	60 Cycles. Power Factor. 95	90	85	80	125 Cycles. Power Factor. 95	90	85	80	Gauge Number, B. & S. or A. W. G.
0000	1.23	1.29	1.33	1.34	1.52	1.53	1.61	1.67	1.62	1.84	1.99	2.09	2.35	2.86	3.24	3.49	0000
000	1.18	1.22	1.24	1.24	1.40	1.41	1.48	1.51	1.49	1.66	1.77	1.95	2.08	2.48	2.77	2.94	000
00	1.14	1.16	1.16	1.16	1.25	1.32	1.35	1.37	1.34	1.52	1.60	1.66	1.86	2.18	2.40	2.57	00
0	1.10	1.11	1.10	1.09	1.19	1.24	1.26	1.26	1.31	1.40	1.46	1.49	1.71	1.96	2.13	2.25	0
1	1.07	1.07	1.05	1.03	1.14	1.17	1.18	1.17	1.24	1.36	1.34	1.36	1.56	1.75	1.88	1.97	1
2	1.05	1.04	1.02	1.00	1.11	1.12	1.12	1.10	1.18	1.23	1.25	1.26	1.45	1.60	1.70	1.77	2
3	1.03	1.02	1.00	1.00	1.07	1.08	1.07	1.05	1.14	1.17	1.18	1.17	1.35	1.46	1.53	1.57	3
4	1.02	1.00	1.00	1.00	1.05	1.06	1.03	1.00	1.11	1.12	1.11	1.10	1.27	1.35	1.40	1.43	4
5	1.00	1.00	1.00	1.00	1.03	1.01	1.00	1.00	1.08	1.08	1.06	1.04	1.21	1.27	1.30	1.31	5
6	1.00	1.00	1.00	1.00	1.02	1.00	1.00	1.00	1.05	1.04	1.02	1.00	1.16	1.20	1.21	1.21	6
7	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.03	1.02	1.00	1.00	1.12	1.14	1.14	1.13	7
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.00	1.00	1.00	1.09	1.10	1.09	1.07	8
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.06	1.06	1.04	1.02	9
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.04	1.03	1.00	1.00	10

When lights only are supplied the power factor will be about 95% as a rule. If mostly lights and a few motors, particularly synchronous motors, the power factor will be 90% to 95%. It will be about 85% if the load is about evenly divided between lights and first-class motors. If motors alone are supplied the power factor will perhaps drop to 80% unless they are well designed and kept well loaded. If the load should be induction motors only partly loaded the power factor may reach 75.

Direct Current Three-Wire Circuits.

Example. — If 500 lamps, 110 volts, one-half ampere, are fed over a line 1000 feet long at a loss of 5%, three-wire system, what size and weight of wire will be necessary?

Solution. —

$$d^2 = \frac{K \times W \times D}{E^2 \times \%} = \frac{2160 \times (500 \times \frac{1}{2} \times 110) \times 500}{\overline{220}^2 \times 5}$$

$$= 122{,}750 \text{ cir. mils.}$$

$$\text{Drop} = \frac{M \times E \times \%}{100} = \frac{1 \times 220 \times 5}{100} = 11 \text{ volts.}$$

$$\text{Weight} = \frac{A \times W \times K \times D^2}{E^2 \times \% \times 10^6}$$

$$= \frac{6.04 \times (500 \times \frac{1}{2} \times 110) \times 2160 \times 500^2}{\overline{220}^2 \times 5 \times 10^6}$$

$$= 370 \text{ lbs.}$$

If the neutral is one-half the size of the outside wires the area would be $\frac{1}{2}$ of 122,750 = 61,370 cir. mils, and its weight would be $\frac{1}{2}$ of $\frac{370}{2}$ = 93 lbs., making the total weight 370 + 93 = 463 lbs. If the neutral is to be the same size as the outside conductors it will add 185 lbs., making 370 + 185 = 555 lbs. If this is the secondary of a transformer system the E.M.F. of each of the two transformers arranged in series to supply the 3-wire line must be 115.5 volts; or if from dynamos direct, they must each deliver 115.5 volts.

Alternating Current, Single-Phase, 60 Cycles.

Example. — Find the size of wire and line drop for the single-phase, 2-wire primary feeders for 1200, 3.5 watt

110 volt lamps, transformers 20 to 1. From generator to transformers 2400 feet. Transformer drop 3%, and in secondary wiring 2%, primary line loss 5% of delivered watts; transformer efficiency 97%.

SOLUTION. —

Power at lamps = 1200 × (3.5 × 16) = 67,200 watts.

$$\text{Power at transformer primary} = \frac{67{,}200}{.98 \times .97} = 70{,}690 \text{ watts.}$$

E.M.F. at transformer primary

$$= (110 + 2\% \text{ of } 110) \times 1.03 \times 20 = 2311 \text{ volts.}$$

Cross section,

$$d^2 = \frac{K \times W \times D}{E^2 \times \%} = \frac{2400 \times 70{,}690 \times 2400}{2311^2 \times 5}$$

$$= 15{,}250 \text{ cir. mils.}$$

This assumes power factor = 95, making $K = 2400$.

The nearest size is No. 8 = 16,500 cir. mils.

Loss % on primary mains, using No. 8, is

$$\% = \frac{K \times W \times D}{E^2 \times d^2} = \frac{2400 \times 70{,}690 \times 2400}{2311^2 \times 16{,}500} = 4.62.$$

$$\text{Line drop} = \frac{M \times E \times \%}{100} = \frac{1.02 \times 2311 \times 4.62}{100}$$

$$= 109 \text{ volts.}$$

Generator E.M.F. = 2311 + 109 = 2420 volts.

Current in primary circuit is

$$I = \frac{T \times W}{E} = \frac{1.052 \times 70{,}830}{2311} = 32.2 \text{ amperes.}$$

SINGLE-PHASE, 125 CYCLES.

EXAMPLE. — Solve the above problem for a 125 cycle circuit.

SOLUTION. — Since M is the only factor which changes with frequency, the solutions and results will be the same for all frequencies except the line drop, and therefore the machine volts.

$$\text{Line drop} = \frac{M \times E \times \%}{100} = \frac{1.09 \times 2311 \times 4.62}{100}$$
$$= 116.37 \text{ volts.}$$

$$\text{Generator E.M.F.} = 2311 + 116.37 = 2428.$$

TWO-PHASE, FOUR-WIRE, 60 CYCLES.

EXAMPLE. — Calculate the line to transmit 3000 H.P. 4 miles to step-down transformer secondaries. Generator supplies such E.M.F. as will give 6000 volts at step-down primaries, and the line loss is to be about 10% of delivered power. Transformer efficiency will be 97% and load of such character as to make the power factor about 85%.

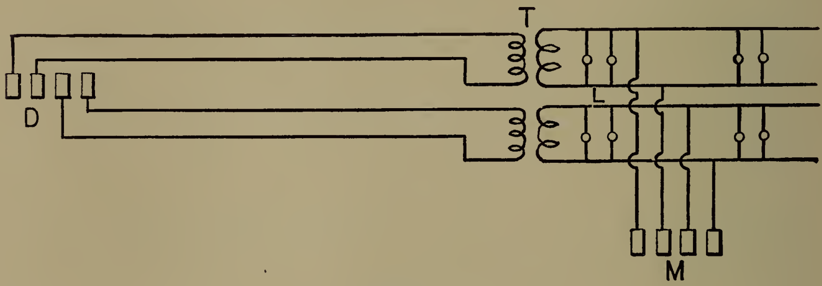

Fig. 41.

SOLUTION. —

$$\text{Power at secondaries} = 3000 \text{ H.P.} = 2238 \text{ K.W.}$$

$$\text{Power at primaries} = \frac{2238}{.97} = 2307.2 \text{ K.W.}$$

Line loss = 10%;

Hence

$$d^2 = \frac{K \times W \times D}{E^2 \times \%} = \frac{1500 \times 2{,}307{,}200 \times (4 \times 5280)}{\overline{6000}^2 \times 10}$$

$= 203{,}040$ cir. mils.

Take 4 No. 3 wires in parallel $= 52{,}600 \times 4 = 210{,}400$ cir. mils.

Loss on mains using 4 No. 3 wires in parallel making 16 wires in all is

$$\% = \frac{K \times W \times D}{E^2 \times d^2} = \frac{1500 \times 2{,}307{,}200 (4 \times 5280)}{6000^2 \times 210{,}400}$$

$= 9.65\%$.

Power lost in transmission $= 3000 \times .0965 = 289.5$ H.P.

$$\text{E.M.F. loss} = \frac{M \times E \times \%}{100} = \frac{1.18 \times 6000 \times 9.65}{100}$$

$= 683.2$ volts.

Generator E.M.F. $= 6683 +$ volts.

$$\text{Line current } I = \frac{TW}{E} = \frac{.588 \times 2{,}307{,}200}{6000} = 226 \text{ amperes.}$$

Add for transformer core loss $1\frac{1}{2}\%$ making 230 amperes.

$$\text{Copper} = \frac{A \times W \times K \times D^2}{E^2 \times \% \times 10^6}$$

$$= \frac{12.08 \times 2{,}307{,}200 \times 1500 \times (4 \times 5280)^2}{\overline{6000}^2 \times 9.65 \times 10^6} = 53{,}675 \text{ lbs.}$$

Two-Phase, Three-Wire Distribution.*

This case may be conveniently worked out by finding first the weight of the copper and size of wire for the single-phase, or two-phase, four-wire system; then when the E.M.F. between the common middle wire and either

* See Crocker's "*Electric Lighting*," Vol. II., page 228, for the derivation of the constants used in this case.

outside wire is the same as in other cases, that is, when the *minimum E.M.F. is the same, the weight of copper in the two-phase, three-wire system will be 73% of that found for the single-phase, or two-phase, four-wire system. Each outside wire will have 42.7% of the area of each in the single-phase, and 85.4% of each in the two-phase, four-wire system. The middle wire will have 60.4% and 120.8% respectively.* In the last example the weight of copper required was 53,675 lbs.

In this system it must be

$$73\% \text{ of } 53{,}675 = 39{,}182.75 \text{ lbs.}$$

The cross section for each outer conductor will be

$$d^2 = 85.4\% \text{ of } 203{,}040 = 173{,}396 \text{ cir. mils.}$$
$$= \text{No. 000 wire, nearest.}$$

The common wire will be

$$d^2 = 120.8\% \text{ of } 203{,}040 = 245{,}272 \text{ cir. mils.}$$
$$= 3 \text{ No. 1 wires in parallel.}$$

When the E.M.F. between the two outside wires is the same as the other systems, that is, when the *maximum* E.M.F. is the same, the rules for the weight of the copper and size of conductors are as follows:

The weight of copper will be 145.7% of that found as directed above. Each outside wire will have 85.5% of the cross section of each in the single-phase and 171% of each in the two-phase, four-wire system. The common middle wire requires. 120.4% and 240.8% respectively.

Assume 6000 volts pressure between the outer wires in the two-phase, three-wire system; the weight of copper

required under the conditions given in last example will be 145.7% of 53,675 =78,204 lbs. The outer wires will have $d^2 = 171\%$ of 203,040 = 347,198 cir. mils. This corresponds most nearly to 2 No. 000 wires in parallel. The middle wire will have 240.8% of 203,040 = 488,920 cir. mils. Take 3 No. 000 wires in parallel.

Three-Phase, Three-Wire Transmission, 60 Cycles.

Example. — Solve the last example for three-phase, three-wire circuits.

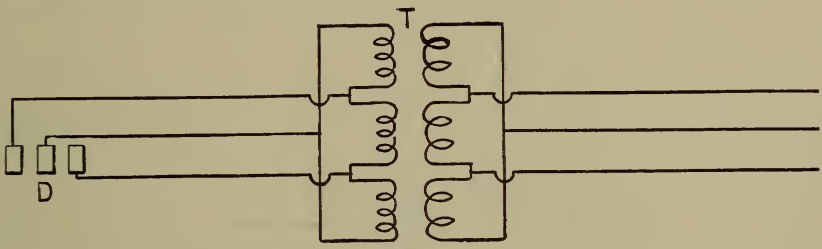

Fig. 42.

Solution. — Power delivered at secondaries, 2238 K.W. At primaries, 2307.2 K.W.

$$d^2 = \frac{K \times W \times D}{E^2 \times \%} = \frac{1500 \times 2{,}307{,}200 \times (4 \times 5280)}{\overline{6000}^2 \times 10}$$
$$= 203{,}040 \text{ cir. mils.}$$

Take 4 No. 3 wires in parallel for each conductor, making 12 in all, giving $d^2 = 52{,}600 \times 4 = 210{,}400$ cir. mils.

$$\% = \frac{K \times W \times D}{E^2 \times d^2} = \frac{1500 \times 2{,}307{,}200 \times (4 \times 5280)}{\overline{6000}^2 \times 210{,}400}$$
$$= 9.65.$$

Power lost on line = 3000 × .0965 = 289.5 H.P.

$$\text{E.M.F. lost} = \frac{M \times E \times \%}{100} = \frac{1.18 \times 6000 \times 9.65}{100}$$

$$= 683.2 \text{ volts.}$$

Generator E.M.F. = 6683 + volts.

$$I = \frac{TW}{E} = \frac{.679 \times 2{,}307{,}200}{6000} = 261 \text{ amperes.}$$

Add $1\frac{1}{2}\%$ for core loss, making $I = 265$ amperes.

$$\text{Copper required} = \frac{A \times W \times K \times D^2}{E^2 \times \% \times 10^6}$$

$$= \frac{9.06 \times 2{,}307{,}200 \times 1500 \times (4 \times 5280)^2}{\overline{6000}^2 \times 9.65 \times 10^6}$$

$$= 40{,}259 \text{ lbs.}$$

Three-Phase, Four-Wire Distribution, 60 Cycles.

Example. — The secondaries from transformers to center of distribution are 500 feet long. The load consists of 5 20 H.P. induction motors, 200 volts, and 1000 16 c.p., 115 volt lamps, $\frac{1}{2}$ ampere each. Allow about 11 volts drop on lighting side from transformers to center, and 5 volts from center to motors. The motor efficiency may be assumed at 80 per cent and power factor 80 per cent. The E.M.F. at distributing center between the main wires is 205 volts. Determine the wiring as in previous examples.

Solution. — For the motors,

$$I_1 = \frac{T \times W}{E \times \text{eff.}} = \frac{.725 \times 4 \times 20 \times 746}{200 \times .80} = 270.5 \text{ amperes.}$$

For the lamps,

$$I_2 = \frac{T \times W}{E} = \frac{.607 \times (1000 \times \frac{1}{2} \times 115)}{200} = 174.5 \text{ amperes.}$$

$$I = I_1 + I_2 = 445 \text{ amperes.}$$

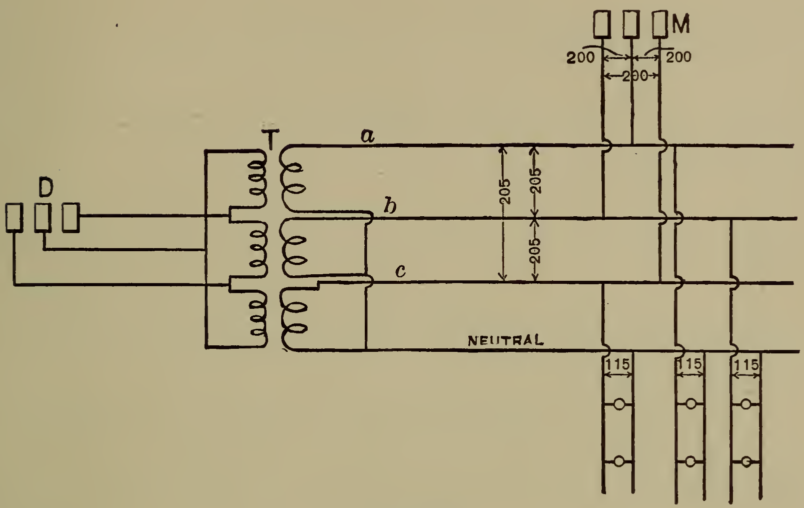

Fig. 43.

For the motors,

$$W_1 = \frac{I_1 E}{T} = \frac{270.5 \times 205}{.725} = 76.5 \text{ K.W.}$$

For the lamps,

$$W_2 = \frac{I_2 E}{T} = \frac{174.5 \times 205}{.607} = 58.9 \text{ K.W.}$$

$$W = 76.5 + 58.9 = 135.4 \text{ K.W.}$$

The drop will be the equivalent of about 6%. Hence

$$d^2 = K \frac{W \times D}{E^2 \times \%} = \frac{1200 \times 58,900 + 1690 \times 76,500}{135,400}$$

$$\times \frac{135,400 \times 500}{\overline{205}^2 \times 6} = 396,500 \text{ cir. mils.}$$

Take 3 No. 00 wires in parallel $= 3 \times 133,079 =$ 399,237 cir. mils.

$$\% = \frac{1200 \times 58,900 + 1690 \times 76,500}{135,400} \times \frac{135,400 \times 500}{\overline{205}^2 \times 399,237}$$

$$= 5.9\%.$$

$$\text{Volts lost} = M\frac{E \times \%}{100} = \frac{1.62 \times 205 \times 5.9}{100} = 19.6.$$

E.M.F. at transformer secondaries $= 205 + 19.6 =$ 224.6 volts.

Drop to center measured between each main and the neutral

$$= 19.6 \times \tfrac{115}{200} = 11.27 \text{ volts.}$$

Cross section of neutral conductor need be only

$$d^2 = \frac{174.5}{445} \times (3 \times 133{,}079) = 155{,}500 \text{ cir. mils.}$$

Use No. 000 = 167,800 cir. mils.

Monocyclic, 60 Cycles.

Example. — The load consists of 4 20 H.P. 110 volt induction motors, 85% efficiency, and 2000 half-ampere, 104 volt incandescent lamps. Distance of transformers from generator, half a mile. Motors are 100 feet from transformers, line loss 4%. Transformer efficiency 97%. Primary line loss 5%. No load E.M.F. of generator 1040 volts. Separate transformers are used for lights and motors. Determine lines as before.

Solution. — At motors

$$W = \frac{4 \times 20 \times 746}{0.85} = 70{,}212 \text{ watts.}$$

For 80% power factor

$$d^2 = K\frac{W \times D}{E^2 \times \%} = 3380 \times \frac{70{,}212 \times 100}{110^2 \times 4} = 490{,}000 \text{ cir mils.}$$

Two No. 0000, or 4 No. 0 wires may be used for each conductor from transformers to motors. The latter will have the advantage of giving the smaller voltage drop for the same loss in watts. The exact proportion of drop will be $\frac{1.28}{1.85} = 69.2\ \%$.

Using No. 0 wires,

$$\% = K\frac{W \times D}{E^2 \times d^2} = 3380 \times \frac{70{,}212 \times 100}{4 \times 105{,}592 \times 110^2} = 4.6\%.$$

E.M.F. drop on secondaries to motors

$$= M\frac{E \times \%}{100} = \frac{1.49 \times 110 \times 4.6}{100} = 7.50 \text{ volts.}$$

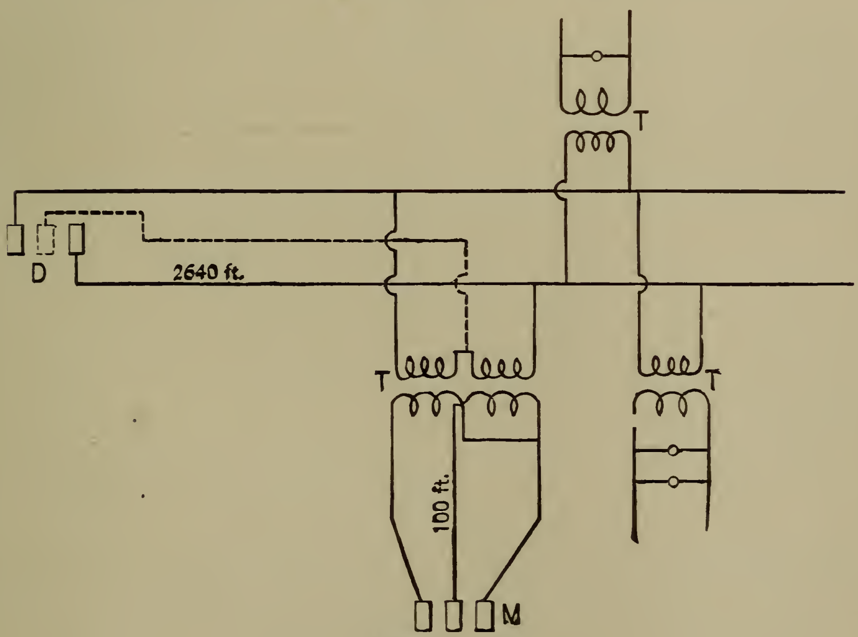

Fig. 44.

Primary E.M.F. at motor transformers

$$= (110 + 7.50) \times 9 \times 1.03 = 1089 \text{ volts.*}$$

* See p. 163 for ratio of transformation.

Hence E.M.F. at secondaries of lighting transformers,

$$= \frac{1089}{10} \div 1.03 = 105.7 \text{ volts.}$$

Also at motor transformers

$$W_1 = \frac{70{,}212 \times 1.046}{(1.00 - .03)} = 75{,}713 \text{ watts.}$$

At lighting transformers

$$W_2 = \frac{2000 \times \frac{1}{2} \times 105.7}{0.97} = 108{,}970 \text{ watts.}$$

$$W = 108{,}970 + 75{,}713 = 184{,}683 \text{ watts.}$$

Total power factor

$$= \frac{108.9 \times .95 + 75.7 \times .80}{184.7} = 88\%.$$

Hence $\qquad K = 2160 \div \overline{.88}^2 = 2424.$

From primary circuit

$$d^2 = K\frac{W \times D}{E^2 \times \%} = 2424 \times \frac{184{,}683 \times 2640}{\overline{1089}^2 \times 5}$$

$$= 200{,}000 \text{ cir. mils.}$$

Take 2 No. 0 wires in parallel, making

$$\% = \frac{2424 \times 184{,}683 \times 2640}{\overline{1089}^2 \times (2 \times 105{,}592)} = 4.8.$$

Primary drop

$$= \frac{M \times E \times \%}{100} = \frac{108.9 \times 1.30 + 75.7 \times 1.49}{184.7} \times \frac{1089 \times 4.8}{100}$$

$$= 72 \text{ volts.}$$

Generator E.M.F. $= 1089 + 72 = 1161$ volts.

$$\text{Compounding} \quad = \frac{1161 - 1040}{1040} = 11.6\%.$$

$$I = \frac{T \times W}{E} = \frac{1.099 \times 184{,}683}{1089} = 186 \text{ amperes.}$$

For teazer wire

$$d^2 = 211{,}184 \times \frac{75.7}{108.9} = 141{,}300 \text{ cir. mils.}$$

Use 3 No. 3 wires in parallel.

Monocyclic, Three-Wire Secondary for Lights and Motors, 60 Cycles.

Example. — From generator to transformer the distance is 2000 feet, and 200 feet from transformer to motors on each circuit, and 200 feet also to center of lights. Drop on secondary mains about 10 volts, on

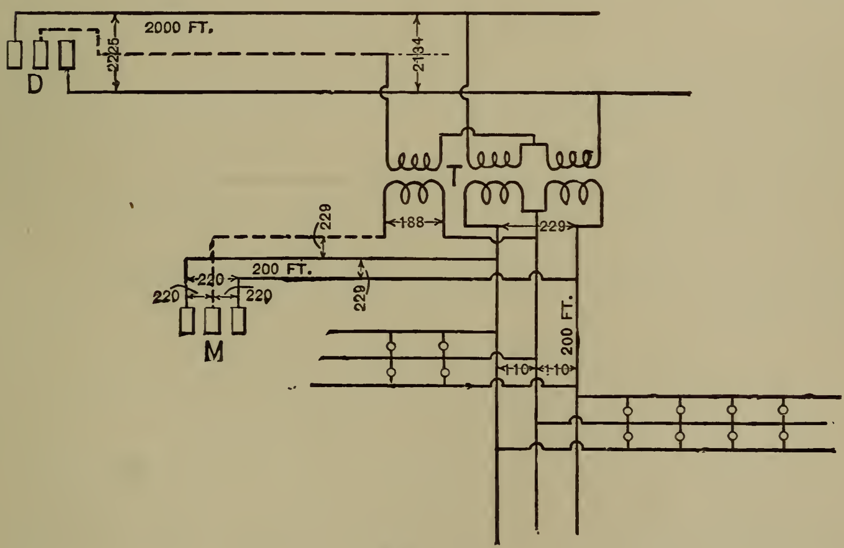

Fig. 45.

primary mains 3.5%, in transformers 3.5%; energy loss in transformers 3%. The load consists of 2000 110 volt lamps and 3 20 H.P. induction motors, 86% efficiency.

Solution. — For lamps

$$W_1 = 2000 \times \tfrac{1}{2} \times 110 = 110{,}000 \text{ watts.}$$

For 10 volts secondary loss

$$\% = \frac{\text{volts loss} \times 100}{M \times E} = 2.8\%.$$

Therefore

$$d^2 = K\frac{W \times D}{E^2 \times \%} = 2400 \times \frac{110{,}000 \times 200}{\overline{220}^2 \times 2.8}$$

$$= 389{,}610 \text{ cir. mils.}$$

Take 3 No. 00 = 3 × 133,079 cir. mils.

$$\% = K\frac{W \times D}{d^2 \times E^2} = 2400 \times \frac{110{,}000 \times 200}{399{,}237 \times \overline{220}^2} = 2.7.$$

$$\text{Drop} = M\frac{E \times \%}{100} = 1.34 \times \frac{220 \times 2.7}{100} = 9 \text{ volts.}$$

Taking neutral = $\frac{1}{3}$ of outside = one No. 00.

E.M.F. at transformer secondary = 229 volts.

In secondary lighting mains

$$I_1 = T\frac{W}{E} = 1.052 \times \frac{110{,}000}{220} = 526 \text{ amperes.}$$

Amount of copper in 1400 feet of No. 00 = 666 lbs.

For motors

$$W_2 = \frac{20 \times 746 \times 3}{0.86} = 52{,}046 \text{ watts}$$

$$= 17{,}348 \text{ watts on each circuit.}$$

In order to make the drop on motor circuits the same as lighting mains for 80% power factor, the % drop must be taken at about 3.5%. Hence

$$d^2 = 3380 \times \frac{17{,}348 \times 200}{\overline{220}^2 \times 3.5} = 70{,}210 \text{ cir. mils.}$$

Take one No. 2 = 66,373 cir. mils.

$$\% = \frac{K \times W \times D}{E^2 \times d^2} = 3380 \times \frac{17{,}348 \times 200}{\overline{220}^2 \times 66{,}373} = 3.6\%.$$

$$\text{E.M.F. drop} = 1.26 \times \frac{220 \times 3.6}{100} = 10 \text{ volts.}$$

This is a little greater than the drop on lighting mains, and may be reduced by taking the next larger size of wire; but for the operation of motors this slight difference is immaterial.

$$I_2 = T\frac{W}{E} = 1.25 \times \frac{52{,}046}{220} = 296 \text{ amperes.}$$

For the total load

$$W = 110{,}000 + 52{,}046 = 162{,}046 \text{ watts.}$$

Total power absorbed by transformer

$$= \frac{162{,}046}{0.97} = 167{,}058 \text{ watts.}$$

Primary E.M.F. $= 229 \times 1.035 \times 9 = 2134$ volts.

For primary feeders

$$d^2 = \frac{110{,}000 \times 2400 + 52{,}046 \times 3380}{167{,}058} \times \frac{167{,}058 \times 2000}{\overline{2134}^2 \times 3.5}$$

$= 55{,}284$ cir. mils.

Take No. 3 $= 52{,}633$ cir. mils.

$$\% = K\frac{W \times D}{E^2 \times d^2} = \frac{2634 \times 167{,}058 \times 2000}{\overline{2134}^2 \times 52{,}633} = 3.67.$$

$$\text{Power factor} = \frac{110{,}000 \times 95 + 52{,}046 \times 80}{167{,}058} = 87\%.$$

$$\text{Primary line drop} = \frac{M \times E \times \%}{100} \times \frac{1.175 \times 2134 \times 3.67}{100}$$

$= 91$ volts.

Generator E.M.F. $= 2225$ volts.

$$\text{Primary current} = 1.14 \times \frac{167{,}058}{2134} = 89.2 \text{ amperes.}$$

$$\text{For teazer } d^2 = 55{,}284 \times \frac{52{,}046}{110{,}000} = 26{,}150 = \text{No. 6 wire.}$$

65. Original Problems. — 1. Neglecting the effect of induction calculate the size of the primary mains and also the secondary leads of a large transformer fully loaded constantly, whose efficiency may be taken at 98%. The secondaries run 200 feet to feed 1000 lamps at 5% loss. The primaries run ½ mile at 5% loss. The ratio of transformation is 20. The lamps are 110 volts. Determine voltages and capacities.

Secondaries a No. 0000 and a No. 000 wire.
Primaries No. 10 wire.
Voltage at secondary terminals 116 volts.
Voltage at primary terminals 2340 volts.
Secondary current 500 amps., primary 2.53 amps.
Machine volts 2463, primary drop 123 volts.
Machine capacity 65 K.W.
Transformer capacity 60 K.W.; lights 55 K.W.

2. A machine of 60 K.W. capacity is to supply two fully loaded transformers at an average of ¾ mile. The secondaries are respectively 200 feet for ⅔ total load, and 100 feet for ⅓ total load. Losses are 5% on secondaries, 5% on primaries. Transformer efficiencies, 97% for the larger, 96% for the smaller. Determine capacity of transformers and lights; also the sizes of primary and secondary wires. The ratio is 10.

Load at transformers 57 K.W.
Secondary, larger, 37 K.W., smaller, 18 K.W.
Load lamps 52 K.W. Primary wire No. 2.
Secondary wire { a No. 0 and a No. 00 in parallel on larger. No. 2 wire on smaller.

3. A load of 1000 incandescent 110 volt lamps is put on the transformer secondaries; the transformers are of nearly one size and are so loaded that about 8% will represent the inductance loss and 1% the resistance loss. The core loss will be 5%. Also 4% line loss is allowed in the secondary wiring. The primary wire is 5 miles long, 18 inches apart, and is No. 00, carrying the current at 125 cycles. Determine the generator volts, transformer pressure, both primary and secondary; also the ohmic and inductive drops. Ratio 10.

Voltage 1100.
Current 50 amperes.
Secondary ohmic drop 44 volts.
Secondary inductive drop 3%, 33 volts.
Transformer ohmic drop 1%, 11 volts.
Transformer inductive drop 8%, 88 volts.
Core loss 5% of 50 amperes = 2.5 amperes.

I (total) = 52.5.

Line ohmic drop $= (0.412 \times 10) \times 52.5 = 216.3$ volts.
Line inductive drop $= (1.23 \times 10) \times 52.5$
$= 645.75 + 15\% = 742.6$ volts.
Total active volts $= 1100 + 44 + 11 + 216.3$
$= 1371.3$.
Total inductive volts $= 33 + 88 + 742.6 = 863.6$.
Total impressed volts $= \sqrt{1371.3^2 + 863.6^2}$
$= 1626$ volts.
Generator apparent watts $= 84365$.
Secondary impressed volts $= \sqrt{1144^2 + 33^2}$
$= 1144.5$ volts.
Primary impressed volts $= \sqrt{1155^2 + 121^2}$
$= 1161.3$ volts.

4. A long-distance plant generates about 1200 volts, steps up to about 12,000, carries the power over 16 miles of No. 1 wire to sub-station, where it is transformed down to about 1000 volts, then carried over wires and distances equivalent to 2 miles of No. 000 wire, then transformed for consumers to 100 volts. The frequency is 60 and the ratio of transformation is 10. Assume a power factor in the secondaries and lamps of 98.5, thus giving an inductance factor of 17. Allow 1% ohmic loss in all transformers and 6% average inductive drop. These per cents are taken on secondary voltages. Core losses are figured at 3%. If the energy delivered is equivalent to 9000 lamps at 50 watts each, make the calculation of pressures and generator energy.

In terms of high voltage line the secondary E.M.F. is 10,000 volts and the current is 50 amperes.

Active secondary E.M.F. = 98.5% of 10,000
= 9850 volts.

Inductive secondary E.M.F. = 17% of 10,000
= 1700 volts.

Step-down transformers,

Ohmic drop 1% of 10,000 = 100 volts.

Ind. drop 6% of 10,000 = 600 volts.

Core loss 3% of 50 amperes = 1.5 amperes.

Total current in primaries = 50 + 1.5 = 51.5 amp.

Primary impressed E.M.F.

$E_i = \sqrt{(9850 + 100)^2 + (1700 + 600)^2} = 10,212$ volts.

Line between step-down transformers,

Ohmic drop = (0.324 × 4) × 51.5 = 67 volts.

Ind. drop = (0.607 × 4) × 51.5 = 125 volts.

Total voltage at secondaries, step-downs,

$E_i = \sqrt{\overline{10{,}017}^2 + \overline{2425}^2} = 10{,}306$ volts.

First step-down transformers,

Ohmic drop = 1% of 10,306 = 103 volts.

Inductive drop = 6% of 10,306 = 618 volts.

Volts impressed on primaries, step-downs,

$E_i = \sqrt{\overline{10{,}120}^2 + \overline{3043}^2} = 10{,}572$ volts.

Current in long-distance line,

$I = 51.5 + 3\%$ of $51.5 = 53$ amperes.

Long-distance line,

Ohmic drop = $(0.65 \times 32) \times 53 = 1100$ volts.

Ind. drop = $(0.65 \times 32) \times 53 = 1100$ volts.

Voltage at secondaries, step-ups,

$E_i = \sqrt{\overline{11{,}220}^2 + \overline{4143}^2} = 11{,}998$ volts.

Step-up transformers,

Ohmic drop = 1% of 11,998 = 120 volts.

Inductive drop, 6% of 11,998 = 720 volts.

Primary impressed pressure,

$E_i = \sqrt{\overline{11{,}340}^2 + \overline{4863}^2} = 12{,}339$ volts.

Primary current from machine,

$I = 53 + 3\%$ of $53 = 54.6$ amperes.

Apparent power = 674 K.W. Lag 23°. Cos 23°
= 0.918, total power factor.

True power = $674 \times 0.918 = 618.7$ K.W.

Total transmission loss

= $618.7 - (9.000 \times 50) = 163.7$ K.W.

Total per cent transmission loss

= $163.7 \div 618.7 = 26.46\%$.

5. Solve problem 5, page 80, by the formulæ of **64.**

6. Determine the wiring for the following single-phase, 125 cycle circuit. From generators to transformers 2000 feet, loss about 5% of delivered power. Transformer drop 3%, energy loss 3%. Drop in secondary wiring 2 volts. Ratio of transformation 10 to 1. Load consists of 1000 16 c. p. 3.6 watt, 104 volt lamps.

Wire No. 3 B. & S.
Primary drop = 68.4 volts.
Generator E.M.F. = 1160.
Primary current = 58.3 amperes.

7. The distance from generator to transformer is 2500 feet, loss 5%. Ratio of transformation 20 to 1; efficiency of transformer $97\frac{1}{2}$%, voltage drop 2%. Secondary loss 2 volts. Load 750, 52 volt 60 watt lamps. Determine the wiring, etc., for a single-phase, 60 cycle system.

Wire No. 3 B. & S.
Primary drop = 54 volts.
Generator E.M.F. = 1155.4 volts.
Primary current = 44.8.

8. Calculate the transmission circuit for a two-phase, four-wire, 60 cycle system to deliver 2500 H.P. 5 miles at 6000 volts, line loss $7\frac{1}{2}$%. Load consists of induction motors. Efficiency of transformers $97\frac{1}{2}$%.

Wire 3 No. 0 B. & S.
Line loss = 574 volts.
Generator E.M.F. = 6574 volts.
I = 199 amperes.
Core loss = 3 amperes.

9. In a two-phase, four-wire system, 125 cycles, 5000 H.P. is transmitted $3\frac{1}{2}$ miles at 10% loss. The E.M.F. at transformers is 5000 volts, and the load has a power factor of about 85%. Determine the size of wire and generator E.M.F.

Wire 4 No. 0 B. & S.
Line loss = 9.79%.
E.M.F. drop = 1042.6 volts.
Generator E.M.F. = 6042.6 volts.

10. Solve 8, assuming a three-phase, 60 cycle transmission circuit.

Wire 3 No. 0 B. & S.
Line drop = 574 volts.
Generator E.M.F. = 6574 volts.
$I = 237.5$ amperes.

11. Solve 9 again, assuming the same conditions for a three-phase circuit, three-wire, but having a frequency of 60 cycles instead of 125.

Line 4 No. 0 wires.
Drop = 646 volts.
Generator = 5646 volts.
$I = 514$ amperes.

12. A three-phase, 60 cycle, four-wire system of distribution is to be constructed to supply a load of 750 115 volt lamps, and 4, 16 H.P., 200 volt induction motors. From transformers to center of distribution the distance is 600 feet. There are to be 15 volts drop on lighting circuits between transformers and center, and five volts from center to motors. The motor efficiency will be about 85%, and E.M.F. at the center about 205 volts.

Lamp circuits 2 No. 0 wires.
Drop = 26.4 between mains, or 15.2 between mains and neutral.
Neutral wire = No. 1 B. & S.
For motors, current = 191 amperes.
For lamps, current = 191.
$I = 392$ amperes.

13. At a distance of 3000 feet from the generator a transformer is stationed to supply 1500 half ampere, 104 volt lamps and one 25 H.P. 110 volt induction motor, efficiency 85%, system monocyclic. From transformer to motor the distance is 100 feet, loss $2\frac{1}{2}$%. Efficiency of transformers 97%. Primary loss 4%. Determine the wiring, etc., as before, frequency 60 cycles.

Motor circuit 2 No. 0 wires.
Drop in motor circuit = 4 volts.
Lighting circuits No. 000 B. & S.
Drop primary circuits = 68.5 volts.
Generator E.M.F. = 1144.5 volts.
$I = 106$ amperes.
Teazer No. 4 B. & S.

14. A load of four 10 H.P. induction motors at a distance of 200 feet from transformers, and 1000, 110 volt lamps, 16 c. p., at a distance of 150 feet from transformers, fed by a three-wire secondary system, monocyclic, 60 cycles, are supplied with power by a generator 1000 feet from transformers. If the primary drop is to be about 3% and the drop in secondary circuits about 10 volts, determine the wiring, etc., assuming transformer drop 4% and energy loss 3%.

For motor circuits, wire is No. 5 B. & S.
Volts loss = 8.25. E.M.F. = 220 volts.
For lights, wire = No. 000 B. & S.
Drop = 8 volts. E.M.F. = 220 volts.
Neutral = No. 2 B. & S.
Primary feeder = No. 7 B. & S. Teazer = No. 8.
Primary current = 48.8 amperes.
Primary drop = 58.5 volts.
Generator E.M.F. = 2192.5 volts.

13. Magnetic Properties (*a*).

Annealed Charcoal Iron.			
B Gausses.	*H* Gilberts per cm. Length.	μ Permeability.	$\frac{nI}{l}$ Amp.-Turns per cm. Length.
3,000	1.34	2,238	1.07
4,000	1.55	2,580	1.24
5,000	1.76	2,841	1.41
6,000	2.03	2,955	1.62
7,000	2.41	2,904	1.93
8,000	2.96	2,703	2.37
9,000	3.56	2,528	2.85
10,000	4.37	2,288	3.50
11,000	5.60	1,963	4.48
12,000	7.60	1,579	6.08
13,000	11.30	1,151	9.04
14,000	16.90	839	13.50
15,000	28.60	524	22.80
16,000	48.80	321	39.00
17,000	100.00	170	80.00
18,000	186.00	96	148.00
19,000	350.00	54	280.00
20,000	650.00	31	520.00

Magnetic Properties (*b*).

Soft Gray Cast Iron.			
B Gausses.	*H* Gilberts per per cm. Length.	μ Permeability.	$\frac{nI}{l}$ Amp.-Turns per cm. Length.
3,000	1.50	2,000	1.20
4,000	5.50	727	4.40
5,000	10.70	467	8.60
6,000	23.20	258	18.50
7,000	43.20	162	34.60
8,000	78.50	102	62.80
9,000	123.70	72	98.90
10,000	188.50	53	150.80
11,000	288.00	39	230.40

14. TRIGONOMETRIC FUNCTIONS.

ANG. DEG.	SINE.	TAN.	COTAN.	COS.	ANG.	ANG. DEG.	SINE.	TAN.	COTAN.	COS.	ANG.
0	0.0000	0.0000	0.0000	1.0000	90	23	0.3907	0.4245	2.3559	0.9205	67
1	0.0175	0.0175	57.2400	0.9998	89	24	0.4067	0.4452	2.2460	0.9135	66
2	0.0349	0.0349	28.6363	0.9994	88	25	0.4226	0.4663	2.1445	0.9063	65
3	0.0523	0.0524	19.0811	0.9986	87	26	0.4384	0.4877	2.0503	0.8988	64
4	0.0698	0.0699	14.3007	0.9976	86	27	0.4540	0.5095	1.9626	0.8910	63
5	0.0872	0.0875	11.4301	0.9962	85	28	0.4695	0.5317	1.8807	0.8829	62
6	0.1045	0.1051	9.5144	0.9945	84	29	0.4848	0.5543	1.8040	0.8746	61
7	0.1219	0.1228	8.1443	0.9925	83	30	0.5000	0.5774	1.7321	0.8660	60
8	0.1392	0.1405	7.1154	0.9903	82	31	0.5150	0.6009	1.6643	0.8572	59
9	0.1564	0.1584	6.3138	0.9877	81	32	0.5299	0.6249	1.6003	0.8480	58
10	0.1736	0.1763	5.6713	0.9848	80	33	0.5446	0.6494	1.5399	0.8387	57
11	0.1908	0.1944	5.1446	0.9816	79	34	0.5592	0.6745	1.4826	0.8290	56
12	0.2079	0.2126	4.7046	0.9781	78	35	0.5736	0.7002	1.4281	0.8192	55
13	0.2250	0.2309	4.3315	0.9744	77	36	0.5878	0.7265	1.3764	0.8090	54
14	0.2419	0.2493	4.0108	0.9703	76	37	0.6018	0.7536	1.3270	0.7986	53
15	0.2588	0.2679	3.7321	0.9659	75	38	0.6157	0.7813	1.2799	0.7880	52
16	0.2756	0.2867	3.4874	0.9613	74	39	0.6293	0.8098	1.2349	0.7771	51
17	0.2924	0.3057	3.2709	0.9563	73	40	0.6428	0.8391	1.1918	0.7660	50
18	0.3090	0.3249	3.0777	0.9511	72	41	0.6161	0.8693	1.1504	0.7547	49
19	0.3256	0.3443	2.9042	0.9445	71	42	0.6691	0.9004	1.1106	0.7431	48
20	0.3420	0.3640	2.7475	0.9397	70	43	0.6820	0.9325	1.0724	0.7314	47
21	0.3584	0.3839	2.6051	0.9336	69	44	0.6947	0.9657	1.0355	0.7193	46
22	0.3746	0.4040	2.4751	0.9272	68	45	0.7071	1.0000	1.0000	0.7071	45
ANG.	COSINE.	COTAN.	TAN.	SINE.	ANG. DEG.	ANG.	COSINE.	COTAN.	TAN.	SINE.	ANG. DEG.

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